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Review

Wire-Grid and Sparse MoM Antennas: Past Evolution, Present Implementation, and Future Possibilities

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Abstract: Since the end of the 19th century, radioelectronic devices (REDs) have actively penetrated into all modern community spheres. Achievements in the fields of radio engineering and electronics, as well as computing, information, telecommunications, and other technologies, have greatly contributed to this. The main elements of REDs are antennas and microwave devices. For example, linear (wire) antennas are the basis of long-distance communication agency networks of various law enforcement agencies and departments. The manufacturing of REDs requires the regular and rapid appearance of more and more advanced types with minimal costs. At the same time, the design complexity of REDs and the tightening of EMC requirements caused by the growth of upper frequencies of useful and interfering signals, the mounting density, as well as the capabilities of generators of intentional electromagnetic impacts, together with the need to take into account inter-element, inter-unit, and inter-system interference, require more and more accurate designs of REDs. However, this becomes impossible without computer modeling, which saves the time and financial resources required for their development, as well as to evaluate the correctness of the proposed technical solutions. During the design process, as a rule, a multivariate analysis or optimization of the product is performed. In this case, methods of computational electrodynamics (one of which is the method of moments) are used. They are based on the replacement of continuous functions with their discrete analogues (construction of a grid), which reduces the problem to the solution of a system of linear algebraic equations (SLAE). The problem's complexity depends on the complexity of the SLAE solution, which is determined by its order (which in turn is determined by the complexity of the simulated object and its surrounding area) and by the number of the required SLAE solutions for each problem (determined by the upper frequency of the signal, the number, and range of the optimized parameters). This dramatically increases the computational cost, which becomes the main constraint for the optimal design. Therefore, reducing the computational cost for the analysis and optimization of RED elements (in particular, linear antennas) is an important scientific problem. Meanwhile, finding new antenna structures that meet all the desired features (low price, required characteristics, manufacturable design with small dimensions and windage, etc.) is no less important today. One of the promise solutions for these problems is using a wire grid and sparse antennas for modeling and constructing antennas. Since the last century, a lot of research has been performed on them. The aim of this paper is to review their history and the main related aspects such as computational, acceleration, and optimization used methods, the fields of their application, and their evolution to this moment. In addition, this paper provides a possible future implementation of wire-grid and sparse antennas from the authors' point of view by presenting a new method that is under research to obtain effective wire sparse antennas.

Keywords: wire grid; sparse antennas; method of moments; computational electromagnetics; numerical methods; antennas; radiation pattern; simulation software

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1. Introduction

One current important trend in electronic technology is to take into account more rigorous system requirements in both commercial and military sectors while maintaining low manufacturing, operating, and maintenance costs. For example, the development of a system capable of working in real time through a special virtual network is relevant to the military sector. Moreover, designing antennas for both ground and onboard subsystems is a unique challenge, as they must be as simple and inexpensive as possible, while at the same time satisfying specific radiotechnical requirements. On the other hand, in the commercial sector, the development of wireless internet access systems, 5G wireless technologies, and the internet of things has led the market to develop low-cost antennas that meet bandwidth, gain, multiband performance, and physical constraints. These trends have prompted developers to create new methodologies and approaches to antenna design. After a long period of separate development of computer-aided design (CAD) systems for antennas and optimization methods, the most promising approach that allows for the identification of design problems and the construction of an optimal structure is the combined use of full-wave electromagnetic modeling and optimization methods. Specifically, the analysis of microwave integrated circuits and printed antennas can be time-consuming, which can lead to a highly inefficient design procedure that typically requires circuit analysis at every iteration of the design cycle, which in turn requires computationally efficient CAD. Thus, it is relevant to develop new methods for synthesizing antenna structures not only to optimize some geometric or physical variables, but also to create the shapes of the structures. These methods should allow for designing an optimal structure that satisfies all the requirements and constraints.

Every decade or so, researchers try to review all the developments and advances in any scientific field. In our time, considerable reviews have been undertaken on modeling and manufacturing antennas and their technologies [1], especially on reconfigurable [2], smart [3], and wearable [4] antennas. In general, these reviews have a narrow focus. For example, this work has been undertaken on beam steering characteristics [5], liquid-based [6], microstrip [7], and switching technology [8] of reconfigurable, meta-wearable [9], smart [10], and transparent [11] antennas. In addition, some studies have been devoted to the design of these antennas where the state of art in designing MIMO [12], 5G [13], and even smart [14] and reconfigurable [15,16] antennas is presented and provides a good database for any interested researcher in this field. However, a general overview of these studies is also helpful, Especially for new researchers. On the other hand, a huge amount of research has been undertaken on modeling antennas, especially those related to modeling wire grid and sparse antennas. Of particular interest are those studies that have used MoM in the past [17–22] and today [23–28], and even those that use other computational methods [29–31]. However, as we know, there exists no extensive general review on modeling wire-grid and sparse antennas using MoM. Therefore, the aim of this paper is to review the issues on modeling antennas using MoM and the main related aspects such as the computational, acceleration, and optimization-employed methods and the fields of their application. In addition, this paper aims to provide a possible future implementation of wire-grid and sparse antennas from the authors' point of view by presenting a new under research method to obtain effective wire sparse antennas.

This paper is organized as follows. In the second section we concentrate on MoM and review the studies of other researchers related to this method and other computational methods. We try to reflect the reasons behind the relevance of investigating MoM. We also provide a considerable amount of research related to the use of MoM and the methods employed to accelerate the modeling and calculating process. In addition, we mention some of the hybridization approaches that used MoM in combination with other methods. In addition, we review some research that contains remarkable experience on using MoM for modeling antennas. In the third section, and as one of the most important aspects when modeling antennas, our review focuses on optimization methods, especially genetic algorithms (GA), and the works that combined MoM and GA when modeling antennas. In the fourth section, we provide a review of reconfigurable and smart antennas, followed by a review in the fifth section of wire-grid sparse antennas, which are important to us. We also present a new method that can be used both for creating and modeling antennas and scatterers and demonstrate its capabilities. This method can be used for manufacturing a sparse wire-grid antenna. The main idea is to avoid using wires with low currents in the antenna grid, since their contribution to radiation is small.

2. Why MoM?

2.1. Overview of Computional Methods

In the past, methods were developed to efficiently solve general electromagnetic (EM) problems, which were based on either differential or integral equations. The resulting systems of linear equations are solved either by direct methods or iteratively. The development of computer technology, combined with the development of fast algorithms with reduced computational complexity and memory requirements, has made it possible to accurately solve problems of electrically large objects numerically. These numerical methods are based on solving partial differential equations (e.g., using the finite difference method [FDM] or finite element method [FEM]) that are reduced to sparse matrices, or solving integral equations that are converted to dense matrix equations (e.g., using the method of moments [MoM]). Solving EM problem with N unknown by a direct solver (Gauss elimination) requires computational complexity O(N^3) and memory requirements O(N^2). However, by correctly ordering the matrix elements, it is possible to effectively compress and solve the corresponding band matrix equation with computational complexity O(N).

The method of conjugate gradients (MCG) [32] is used to solve the Poisson equation. Thus, it converges in O($N^{0.5}$) iterations for two-dimensional problems and in O($N^{0.33}$) for three-dimensional problems, respectively. When a multigrid method is used to solve the same equation, the number of iterations does not depend on the order of the matrix [33]. As a consequence, the total computational cost associated with the MCG for such problems scales as O($N^{1,5}$) for two-dimensional problems and O($N^{1,33}$) for three-dimensional problems. For multigrid methods, the total computational cost scales as N.

The finite difference time domain (FDTD) method is one of the most popular methods for solving electrodynamic problems numerically and was first described in [34]. However, the abbreviation of the method's name was suggested by the author of [35]. The method is based on a procedure deployed in the time domain during which continuous electromagnetic waves are replaced by their discrete analogues [36]. The discrete representation is realized for the space and time included in Maxwell's equations. This method is a simple and straightforward approach to solving Maxwell's equations and has an explicit solution scheme that does not require linear algebraic equations (SLAEs) to be stored, as well as an implicit scheme that reduces to a system of SLAEs [37]. Besides, the FDTD method quite simply describes the inhomogeneity of the medium. Working in the time domain allows for results to be obtained in a wide range of frequencies. This method allows the researcher to specify the material at each point of the volume and choose not only from a wide range of metals and dielectrics, but also materials with nonlinear properties. It allows one to directly simulate the effects on the holes as well as the shielding effects, while the fields inside and outside the shield can be calculated either, directly or not. However, this method, like all methods, has disadvantages. First, the counting volume must be divided into a very large number of cells (the subarea [cell] size is small compared to the smallest wavelength), which means large memory and simulation time costs. This makes it difficult to simulate problems with long, thin spatial structures, such as the fields obtained from currents along conductors. Second, the calculation of the fields is performed at each point of the counting volume, so if you want to find the field at a certain distance from the source, it probably means that the counting volume will be excessively large. But it must be finite to fit in the computer's memory, which in most cases is achieved by setting artificial boundary conditions on the counting volume, the use of which can lead to data distortion. Nevertheless, in the case of using FDTD to solve the wave equation directly in the time domain, the computational complexity is the same as the MCG ($N^{1.5}$ for the two-dimensional problem and $N^{1.33}$ for the three-dimensional problem) [38], except that FDTD generates a solution for the entire time domain, and therefore for all frequencies at once. This algorithm is optimal because it generates O(N^x) solutions using O(N^x) operations.

The Finite Element Method (FEM) is widely used in the mechanical analysis of structures. Despite the fact that the mathematical treatment of the method was proposed in 1943 by Courant [39] for solving electromagnetic problems, this method was not applied until 1968. Since then it has been used in the analysis of waveguides, electrical machines, semiconductor devices, multiconductor lines, electromagnetic radiation from biological objects, etc. [40–47]. The general nature of the FEM allows for the building of universal computer programs based on it in order to solve a wide range of problems. Therefore, programs developed to solve problems from other disciplines can be and have been successfully applied to solve problems from another discipline with or without minor modifications [48]. The main idea of FEM is that any continuous value (temperature, pressure, etc.) is approximated by a discrete model that is constructed using a set of piecewise continuous functions defined on a finite number of subdomains [49]. The algorithm for constructing a discrete model of a continuous quantity is as follows. A finite number of points are fixed in the area under consideration. These points are further called nodes. Assume that the studied continuous value at each node is a variable that is to be determined in the process of solving the problem. The region of the continuous value change is divided into elements. These elements have common nodes between them and, together, approximate the shape of the entire region. The continuous value is approximated within each element by a polynomial, the coefficients of which are calculated according to the quantities of this value in the nodes. Each element is approximated by its own polynomial, and the coefficients of the polynomials are chosen so that the continuity of the value along the boundaries of neighboring elements is preserved. Then, the finite elements are combined into an assembly. In this assembly, the nodal quantities of the unknown functions are chosen in such a way that a sufficient approximation of a continuous distribution is achieved. This step leads to a SLAE with respect to the nodal quantities. After that, the resulting system has to be solved, i.e., the nodal quantities must be found. To approximate a continuous region, when solving the formulated boundary problems, a system of pyramidal elements is used that allows writing down a system of equations for E (or H). Their solution determines the interpolation polynomial coefficients. Then the finite elements are combined into an assembly, and the interpolation functions are expressed through global nodal quantities and global coordinates (related to the model as a whole). A sufficiently complete presentation of the method is given in [50,51]. The main advantages of FEM are the clear physical meaning at all stages of the calculation, which allows one to easily control the results, and the simplicity of calculating the characteristics of combined structures that include multiple elements of various sizes. The disadvantages of FEM lie in the need for an efficient finite element generator; the complexity of the equations; the discretization of the occupied region with a variable step, which leads to the increase in the computational cost while forming a volumetric model; and the increase in the computational cost in multicomponent environments. If the FEM nodes are ordered correctly, the dense matrix will appear only in the lower right corner of the system matrix [52,53]. In this way, it is possible to find the inversion of the system matrix using the matrix splitting method. When the nested dissection ordering [54] is applied to the sparse part and the LU-decomposition is applied to the dense part, the total computational complexity is O($N^{1,5}$) for two-dimensional problems and O(N^2) for three-dimensional ones. The memory requirements are O($N\log N$) for two-dimensional and O($N^{4/3}$) for three-dimensional problems.

One of the disadvantages of differential equation solvers is the mesh dispersion error that causes the wave to have a different phase velocity within the mesh compared to the exact solution [55,56]. This error can be eliminated by using a higher mesh density but causes an increase in the computational cost. Since the error is cumulative, it is especially noticeable when simulating a large region or in the case of large scatterers. To eliminate this error, the density of the mesh must be increased as the size of the modeling region increases. For second-order precision designs, the mesh density in one dimension (number of points per wavelength) is defined as $(kd)^{0.5}$, where d is the diameter of the modeling region, and k is the wave number [57]. Consequently, the number of unknowns is $(kd)^{1.5}$ for one dimension. Correspondingly, for two dimensions the number of unknowns scales as $(kd)^3$, while for three dimensions it scales as $(kd)^{4.5}$. The solution lies in using an accurate higher-order differential equation solver [58,59] or in combining a differential equation solver with an integral equation solver when there are large homogeneous regions.

Among the existing numerical methods, MoM [60] is one of the most frequently used methods in modeling microwave and millimeter integrated circuits and is widely applied to the problems of electromagnetic compatibility [61]. The name "method of moments" is, according to some authors, unfortunate, because it has a slightly different meaning in modern applied mathematics [62]. However, when solving the problems of electromagnetism, this approach is historically still called MoM. When Harrington chose a name for the method he used, he borrowed it from the work [63] where the authors described a method for solving an integral equation and called it MoM. Earlier, a method with this name was proposed in [64] where polynomials (moments functions) were used as test functions for solving integral equations. However, it should be noted that within Harrington's paper, he used the term "method of moments" to refer to the method of weighted residuals (MWRs), despite the fact that he limited himself only to linear electromagnetic problems [65]. In the version of MoM proposed by Harrington, piecewise constant (impulse) functions are used as basic functions and Dirac functions as test functions. This is also known as the collocation method. Harrington in [66] made a generalization that allows us to consider the collocation, Galerkin, and least-squares methods as special cases of MoM. According to some authors, FEM and even FDM methods can also be considered as special cases of MoM [67]. In applied mathematics, however, this approach is commonly referred to in another way. Thus, in 1956, the work [68] proposed the term "weighted residuals", which generalizes a whole family of these methods. A general discussion of this approach can be found in [69,70]. The development of the MWRs is manifested in the works of [66,71–75].

2.2. The Use of MoM

In MoM, the boundary value problem for the unknown current distribution over the conductor surface is often formulated as an integral electric field equation. Then, applying suitable basic and test functions, these equations can be transformed into a SLAE with a dense matrix. The computational complexity of solving a SLAE of order N is $O(N^3)$ by the direct method. When modeling millimeter-scale integrated circuits where N is fixed and relatively small, it is sufficient to use a traditional direct method for solving the SLAE. After solving it, circuit characteristics, such as S-parameters, radiation losses, etc., can be obtained from the current distribution. These can be obtained by applying MoM either in the spectral domain [76] or in the spatial domain [77].

Over time, this method has been used and developed considerably. For instance, [78] presents a method based on the Galerkin realization and is closely related to the approach in the spectral domain. The results are a full harmonic/time electromagnetic analysis of the microstrip, which can be used to evaluate individual microstrip inhomogeneities or, on faster computers, the entire microstrip circuit. The proposed approach is quite effective, since the results of the analysis of simple circuits can be obtained in a reasonable period of time, even by using a personal computer. Then, in [79] the authors also used MoM to solve the integral equation for an accurate full-wave analysis of shielded microstrip inhomogeneities, obtained by applying the reciprocity theory. This paper [80] presents the results of full-wave analysis in the spectral domain using the Galerkin method with vector functions of a triangular subarea as expansion and testing functions of the MoM procedure to investigate various microstrip inhomogeneities. This approach proved itself to be a very accurate method for analyzing compensated microstrip inhomogeneities. A grid structure and database for storing impedance matrix elements that can be used to analyze structures of different geometries are demonstrated. MoM is also used in [81] to solve an integral equation reduced to SLAE, expressed in a modified method of spectral analysis proposed by the authors to calculate three-dimensional microwave structures. Based on the realization of MoM in the spatial domain, a full-wave version was developed for analyzing and determining the circuit characteristics of passive microstrip elements (in an open environment) at microwave frequencies [82].

2.3. MoM Acceleration

Computation time and memory requirements are greatly reduced by efficient moment matrix filling, the use of closed-form Green's functions, and symmetry in the problem formulation; thus, a circuit of moderate electrical size can be analyzed in an acceptable period of time on a personal computer. The solvers of integral equations usually use fewer unknowns than the solvers of differential equations, because in the first case only the induced sources are unknowns, whereas in the latter case the unknown is the field. However, the use of integral equation solvers results in dense matrices [83]. If the matrix equation is solved using the LU-decomposition (Gaussian elimination) or the iterative method (CG or similar methods), the computational cost can be significant [84]. Performing the LU-decomposition requires $O(N^3)$ operations and $O(N^2)$ memory and provides a solution for all scatterer excitations. The CG method requires $O(N^2)$ operations per iteration for dense matrices, since the costliest step in the CG iteration is the multiplication of the matrix by a vector. As a rule, the number of iterations increases as the electrical size of the object increases.

Many researchers have tried to reduce the complexity of the traditional MoM algorithm by reducing the computational cost of the corresponding matrix-vector multiplication. Methods that use matrix operations, such as the order recursive method of moments (ORMoM) proposed in [85] or the fast multipole method (FMM) proposed in [86], reduce the computational cost of matrix multiplication. The impedance matrix localization (IML) method developed by [87] is based on the use of basic functions that lead to a sparse MoM matrix, which, in turn, speeds up the operation of matrix multiplication by a vector. It is worth noting that IML is only suitable for smooth surfaces. Likewise, the complex multipole beam (CMB) method [88] was introduced, which is also applicable only to smooth surfaces. The pseudo-grid algorithm described in [89] has been used to develop a mixed potential integral equation method in the spatial domain. This method, which may be used to analyze microstrip inhomogeneities and antennas of arbitrary shape, is derived from applying MoM to the mixed potential integral equation (MPIE) in the spatial domain to find both current and charge distribution on the microstrip surface. The ordered recursive Gaussian elimination (ORGE) method [90] effectively exploits the data overlap that occurs when designing and optimizing millimeter-wave integrated circuits (MMICs) using MoM, because multiple subtasks involving dimensional changes in different parts of the circuit must be iteratively modeled, which leads to significant data overlap among these subtasks since each subtask is solved independently of the others, without regard to data overlap. Later, researchers proposed the direct matrix manipulation (DMM) in [91] and methods based on wavelets [92], then in [93] they proposed a multistructure MoM technique based on simultaneous analysis of different structures. In practice, all structures under study are derived from the same global structure, and solutions of derived problems are obtained by simple analytical manipulations performed on the solution of the global structure. This concept avoids a large number of repetitive calculations and therefore dramatically reduces the computation time. Unlike most other fast methods, this method does not rely on simplifying assumptions but simply reuses the available information for further calculations. As a result, the proposed simultaneous simulation provides exactly the same accuracy as the equivalent individual simulation. Meanwhile, the computer requirements are significantly reduced.

The work on reducing the modeling time is mainly related to obtaining an approximation of the original matrix with acceptable accuracy. This approximation is used to speed up the iterative solution by substituting the original matrix with the approximated one. The main results on the theory of dense matrix approximation were obtained by the research group of E. Tyrtyshnikov. During the work under the grant RNF 14-11-00806 (2014–2016), the authors, in particular, investigated the application of algebraic methods of approximation of large dense matrices to increase the computational efficiency of methods for solving integral equations of mathematical physics. Moreover, in the work [94], the researchers developed and implemented algorithms for constructing mosaic-skeleton approximations, the iterative method of GMRES in relation to the solution of SLAEs with an unchanged matrix in mosaic-skeleton format and several right-hand sides. The developed algorithms are applied to solving problems of scattering of electromagnetic waves on ideally conducting surfaces of complex shape.

Ilyin's works are devoted to the solution of very large sparse SLAEs obtained by approximating the boundary value problem by means of a finite-volume exponential scheme, with the help of new methods under development such as the method of bisconjugate residuals, the dual method of bisconjugate residuals, and the stabilized method of bisconjugate residuals. Thus, the work [95] considers a set of algorithmic and technological problems related to the development, research, and application of high-performance parallel methods for solving large SLAEs with sparse matrices. A number of the presented original results are related to the development of iterative processes in the Krylov subspaces, as well as principles of their preconditioning and scalable parallelization based on additive domain decomposition algorithms. The concept of the Krylov library is described as integrated open software for a wide range of linear algebra problems.

There are ideas of developing iterative methods for solving sparse SLAEs [96,97] and employing the approximation of the original dense matrix, not only to reduce the required memory size, but also to speed up the iterative process by using the approximated matrix as a preconditioner. The main results of improving on E. Tyrtyshnikov's ideas were obtained by S. Rjasanov and M. Bebendorf [98]. In work [99], the method of moments in the frequency domain was applied to model electrically large problems. To speed up the computations, a parallel version of LU decomposition was used, and the unknowns were grouped into blocks that were approximated by matrices of small rank using adaptive cross-approximation. The proposed method was tested on problems with the number of unknowns reaching four million.

In the work [100], two block iterative methods for solving sparse SLAEs obtained by finite difference and element methods were proposed. Their computational costs for solving SLAEs with a constant matrix and several right-hand sides were considered. The effectiveness of the proposed methods was demonstrated by the example of electromagnetic analysis of two test three-dimensional structures. It as shown that when using an effective preconditioner, it is preferable to use block methods to sequentially solve all SLAEs with different vectors of free terms. In the work [101], new methods for solving a number of SLAEs with a shift (with matrices differing from the original matrix by a scalar multiplied by a unit matrix) and many right-hand sides were proposed.

2.4. MoM Hybridization

Each method is applied to the problem region for which it is best suited. The appropriate boundary conditions are applied at the borders between these regions. The method of boundary integral equations is used in combination with MoM to determine, based on the analysis of frequency-dependent lossless and lossy propagation, the characteristics of open multiconductor transmission lines in multilayer media [102]. A complete procedure for optimizing microwave and millimeter-wave planar circuits using MoM is presented in [103]. In this procedure, a new concept of electromagnetic optimization using MoM was proposed. It is based on the study of the smallest independent units of information, the so-called elementary invariant operands.

None of the numerical methods are suitable for all required electromagnetic modeling problems. For example, MoM program codes are practically unsuitable for describing inhomogeneous nonlinear dielectrics. Finite element codes cannot effectively model large scattering problems. Multipole and diffraction-based approaches are not suitable for small complex geometries or problems that require accurate determination of the surface current. Unfortunately, there are tasks where all these features must be taken into account, for example, when evaluating radiation from a printed circuit board, and therefore, the analysis cannot be performed by any of these methods. There are some hyper-methods that result from combining MoM with other methods. One of the solutions is to combine two or more methods in one program code. Moreover, one of these methods is often MoM. For example, a hybrid method, which combines two numerical methods: the finite-difference formulation of alternating-direction implicit (ADI) in the time domain (ADI-FDTD) and the method of moments in the time domain (MoMTD), was proposed to simulate a short-pulse ground-penetrating radar (GPR) [104]. Another hybrid method based on the equivalence principle is presented in [105]. Additionally, the work describes the problem of antennas located on the ground. This problem has been split into two related equivalent problems: one for the antenna geometry and one for the ground geometry. The fields in each region can be modeled using the most appropriate numerical methods. FDM in the time domain was used for modeling the surroundings because it is well suited for modeling fields in inhomogeneous media, while MoM was used for antennas because it is well suited for modeling complex antennas in free space. The same hybrid approach (the combination of MoM and FDM in the time domain) was also applied to estimate the specific absorption ratio (SAR) of a rat inside a reverberation chamber (RC) [106]. This hybrid approach was an alternative method used to solve the problem of poor FDM convergence when analyzing the RC. Initially, RC with a dipole or helical antenna was numerically designed to operate at a frequency of 2 GHz. The MoM/FDM method was then used to calculate the average whole-body SAR (WBA-SAR) of a small animal inside the chamber case. The results were compared with those obtained using the FDM method to verify their accuracy. The combined MoM/FDM approach was also used to analyze the effects of millimeter waves on a rabbit's eye, using a dielectric lens antenna as the source of electromagnetic emission [107]. MoM was used and validated by comparing the calculated and measured electric fields emitted by the dielectric lens antenna. The hybrid method was then used to determine the specific absorption coefficient of the impact of millimeter waves on the eye of a rabbit placed in the focus of the antenna. In 2007, a highly efficient computational method based on a hybrid formulation of the physical optics moment method (MM-PO) combined with impedance matrix interpolation and dynamic adaptive frequency discretization was presented for broadband analysis of antennas radiating in the presence of conducting objects [108]. The author then used this technique for broadband analysis of antennas radiating in the presence of electrically large conducting bodies (platforms). The results of using this method are presented, and they showed significant savings of both computer memory and CPU time [109]. In 2011, MoM was comprehensively hybridized with other methods (e.g., FEM, PO, or UTD) and accelerated with the multilevel fast multipole method (MLFMM) [110]. A technique for combining the FEM solution for volume electric fields and MoM (or boundary integral solutions for the cut-off boundary) to solve open boundary problems was presented in [111]. This technique was based on a highly parallelized domain decomposition infrastructure.

2.5. MoM Antennas

Historically, MoM has been developed very intensively in relation to the problems of electrostatic and electrodynamic analysis of wire structures [112,113]. Their analysis is an important task because they are used as antennas [114–118] in grounding and equipotential bonding systems, in surface approximation [119,120], and in the development of various electromagnetic field simulators [121]. The analysis of linear (wire) antennas is reduced to the solution of the integral (integro-differential) Pocklington [122] and Hallen [123] equations. The specific features of the solution of these equations are based on the thin-wire approximation and are discussed in detail in several works, e.g., [124–128]. In this approach, the conductor is assumed to be an ideal conductor in the form of a cylinder located along one of the coordinate axes (one-dimensional problem), with a cross-sectional radius much smaller than the wavelength of the excited signal and the conductor's physical length. It is also assumed that the current is zero at the ends of the wire. This simplification allows one to use a scalar current density function instead of a vector function, which significantly reduces the complexity of the problem.

The time required to solve the problem became the determining factor in choosing one or another numerical method [129]. In addition, the required machine memory capacity is important, as it allows the use of relatively weak workstations to solve complex problems [100]. The use of methods for solving SLAE is also strictly associated with the peculiarities of storing its matrices [130–132]. Reducing the time to form and solve the SLAE to reduce the computational cost of modeling, for example when using MoM, can be achieved by choosing the method of structure discretization, which allows approximating its regions with a smaller number of subdomains, thereby reducing the order of SLAE [133]. In this case, the selection of the sets of basis and test functions most suitable for each specific task is the most important [134]. The choice of basis and test functions, which allow approximating the desired function with a small number of them [135], reduces the time spent on calculations. The choice of similar basis and test functions gives a symmetric matrix [136]. To accelerate MoM, solid-state storage devices (SSDs) [137] and parallel computing on the CPU and/or GPU [138] were also used.

Approximation of the SLAE matrix can be achieved by using iterative methods [139], low-rank approximation of matrices [140], zeroing (discarding) the matrix elements of the generated SLAE, whose values are less than a given threshold, and, consequently, transforming the dense matrix into a sparse one, with its further solution by one of the methods for sparse systems [141]. As a result, a general research trend is the reduction of the time required to calculate with the required accuracy and to obtain an optimal structure.

Currently there is a growing interest in the optimization of electromagnetic devices. As a consequence, significant efforts have been made to develop competitive optimization methods. However, it should be noted that different optimization algorithms may be used for different applications in order to meet the specifications of the problem [142]. It is also worth noting that the geometric shape plays an important role in changing the characteristics of the design. For example, for a patch antenna, the circular polarization characteristic can be obtained either by placing stubs in proper locations or by placing diagonal connectors in the center of the antenna. In addition, the antenna can be tuned using a powered configuration, or the bandwidth can be increased by adding parasitic elements. It is outstanding that for different types of applications, only a small part of the antenna structure is changed; it is either removing from or adding to the structure's geometry to obtain the desired specification. Consequently, there is no need to solve the entire problem at each iteration in the optimization process. This is because in each iteration step, only a few rows and columns are added or removed from the matrix system used in the previous iteration step. Since this procedure greatly reduces the computation time, it becomes possible to evaluate, in real time, the effects caused by the changes in geometry that are made during the design process to improve system performance.

3. Optimization

3.1. Optimizing Methods

It is very important to choose the optimization algorithm that is best suited to the task being solved. The goal of optimization is to minimize (or maximize) the objective function under certain constraints. Optimization using the directed search approach can be achieved by using an iterative algorithm consisting of two steps. In the first step, the "improvement" direction is determined at the iteration point, followed by a one-dimensional search in which the point of the next iteration is calculated by finding the minimum in the "improvement" direction that was determined earlier. Among different types of directional search algorithms, the most common are the steepest descent (SD), Newton, CG, and quasi-Newton methods [143]. As a rule, SD is the first choice in many applications because of its simplicity and globally convergent behavior. However, in many other applications that require a large number of variables, CG and quasi-Newton methods are preferred, which also globally converge, because of their better performance, especially at the optimal points. The most commonly used one-dimensional search algorithms are the equal interval search, golden ratio search, dichotomous search, Fibonacci search methods, and parabolic fitting method [144]. If a fixed interval of uncertainty with a minimum number of function estimates is required, the Fibonacci search algorithm is the preferred choice. However, the genetic algorithms (GA) [145] and simulated annealing methods [146], which are stochastic rather than deterministic in nature, represent two alternatives to the traditional optimization algorithms. In addition, they use a directed random search to find optimal solutions in complex domains.

3.2. GA

Since traditional optimization algorithms deal with local properties of the iteration points, they may stall at a local extremum. In contrast, GAs use a random search in addition to a systematic search, which prevents these methods from hitting a local minimum or maximum. In fact, it was proved that under certain conditions, Gas converge to the global optimum [147]. Gas are structured to solve real-world problems by simulating processes that occur during natural evolution. The coding mechanism maps each solution as a unique binary string. The set of strings constitutes a population, and it evolves from generation to generation through the use of genetic operations, the most common of which are reproduction, crossover, and mutation [148]. The number of function estimates in one generation depends on the generation base used for optimization. For conventional Gas, determining the population size depends on the specific task and is highly dependent on chromosome length and power [149]. When applying Gas, most of the steps can be performed in different ways, and there are many parameters that require precise tuning. Consequently, it may be necessary to test different combinations of possible parameter choices and possible ways of applying genetic operators in order to achieve the maximum benefit from GA for each task. The concept of the genetic algorithm, first formalized in [150] and extended to functional optimization in [151], involves the use of optimization search strategies modeled after Darwinian notions of natural selection and evolution. Many extensions of and improvements to the simple GA optimizer have been developed and used. Among them are the concepts of elitism [152], the use of steady-state algorithms [153], the use of real coded parameters [154], the concepts of community-based genetic algorithms [155], and approaches applicable to the traveling salesman problem.

3.3. The Use of GA

The application of modern electromagnetic theory to radiation and scattering problems often requires the use of optimization. Among typical problems that require optimization are reflector design [156], object resonance extraction [157], and the development of an anti-reflective coating for a low radar cross section (RCS) [158]. Other tasks, such as the formation of the antenna array radiation pattern [159], which were solved without the use of optimization, are often easier to solve with optimization. This applies, in particular, when dealing with implementation limitations imposed by production considerations or environmental factors, differentiated regions, and/or discontinuous regions. In such situations, approximations or models of true electromagnetic phenomena are often used to save computational resources. These characteristics severely test the capabilities of many traditional optimization methods and often require their hybridization. For example, the use of GAs has become very common because of their special characteristics that make them an ideal tool that combines well with existing EM analysis methods and usually gives satisfying results. GAs are well suited for tasks where one needs to find an optimal solution among a large number of admissible ones. Consequently, in addition to the discreteness of the problem due to the large search space, GAs show good results when applied to this type of optimization problem. GAs are significantly more efficient and provide much faster convergence than random walk algorithms. In addition, they are easy to program and easy to implement. In contrast to gradient search, GA optimizers can easily handle discontinuous and undifferentiated functions.

The use of evolutionary optimization strategies for EM design has also become widespread. Thus, in [160], a new method for optimizing the design of electromagnetic devices that uses GAs as a search method was presented. The method is used to optimize the pole shape of an electric motor. The electromagnetic analysis of the implemented devices is performed using FEM. The paper [161] presents an example of GA used for electromagnetic optimization. This work describes the development of lightweight broadband microwave absorbers, the design of antenna arrays with shaped-beam patterns, and the extraction of resonant modes of radar targets from backscatter response data. In addition, the paper [162] describes the basics of GAs, presents their history regarding their EM applications, describes the application of modern genetic operators to the field of electromagnetism, and presents the design results for various applications. The paper [163] presents a new optimization technique that combines classical and statistical methods in an innovative and efficient way. In particular, an evolutionary GA that uses a local minimization scheme based on the conjugate direction method was developed. The proposed optimizer can be applied to the design of planar microwave circuits and printed antenna arrays. The paper [164] presents the results of experiments to determine the optimal population size and mutation rate for a simple genetic algorithm. It was shown that the use of a small population size and relatively high mutation frequency is much preferable to the large population size and low mutation frequency used in most articles.

The proliferation of wireless communication networks has created a natural interest in network optimization. In particular, the emphasis on low cost in many modern wireless networks often imposes severe limitations on the acceptable level of complexity of the transceiver processing circuitry. The available transmitter power in modern networks is also often high. Therefore, solving the design problem usually requires significant optimization in conjunction with network modeling. Using GAs, the authors in [165] managed to maximize the nodal signal-to-noise ratio (SNR) while minimizing transmitter power levels. The GA approach to adaptive phase zeroing is significantly faster than random search and gradient methods, which was proven in the work [166] and later in [167] as applied to satellite communication antennas. The GA was developed to optimize the number and size of cells for a circular symmetric grid. Moreover, numerical optimization based on GAs has been used to develop multilayer broadband radar absorbers [168]. In [169], a procedure for synthesizing radio-absorbing materials to reduce the radar cross section in the broadband frequency range is presented. GAs have also been widely used to optimize microstrip circuits, as in [170,171].

3.4. Antennas and GA

When designing antennas, there may be a need for a particular desired shape or desired multiband or even multifrequency operation. Optimization methods find application in this important case as well. In 1994, the authors of [172] proposed the use of GA in antenna design. Later in 1996, a deductive antenna design method was proposed to specify the desired electromagnetic properties, resulting in the so-called genetic wire antennas [173]. The authors of [174,175] then developed this approach and applied it to synthesize wire antennas loaded with concentrated components. In 1999, this combination was used to simultaneously control both achievable gain and grating lobe reduction in the optimal synthesis of Luneberg lens antennas in [176–178]. In [179], GA was also used to optimize the maximum level of the side lobes of a planar array. Under the same work title, researchers used GA for amplitude and phase adaptive zeroing in [180], and the same has been undertaken later in [181]. The same optimization method was used to design automobile antennas in [182], patch antennas with circular polarization in [183], and uniformly excited broadband low sidelobe linear and planar antenna arrays in [184]. A new application of GAs to design rectangular microstrip antennas was presented in [185–189]. Using FDM and GA, the authors of [190] designed a compact broadband low-profile antenna. In [191], GA was used to synthesize a directional circular arc array pattern to minimize the bit error rate in an indoor wireless communication system. In [192], optimized shapes of multiband microstrip antennas using GA were investigated. For two-, three-, and four-band microstrip shapes, the obtained antennas showed good performance at the design frequencies.

3.5. MoM Antennas and GA

MoM has proved to be the best option among a large number of numerical methods used in combination with one of the optimization methods to improve the matrix-filling procedure and reduce the solution time. Thus, there have been attempts to combine Gas and MoM, for example. In particular, the paper [142] describes the design of compact microwave filters using GA in combination with the 2.5D MoM. In addition, the paper [193] presents the results of applying well-known numerical simulation codes (NEC) and GA to design wire antennas with circular polarization and wide angular coverage. In both cases, the results obtained from GA optimization showed excellent performance for the chosen optimization parameters. The method of creating new shapes for resonant structures is presented in [194]. To analyze the tested structures, the full-wave method in the spectral domain was used. This method effectively uses GA to find new forms of resonant structures. Meanwhile, the authors of [195] demonstrated the possibility of creating new, fully dielectric waveguide grating filters using GA to optimize the dielectric material permittivity and placement of geometric boundaries that separate homogeneous regions. The work [196] describes a technique to effectively combine GA and MoM to design an integrated antenna. This technique was proposed earlier in [197], and it is based on using the direct Z-matrix transformations (DMM) for effective GA and MoM integration. Using DMM with GA/MoM significantly reduces the overall optimization time by eliminating several filling operations on the Z-matrix.

As an extension of an effective numerical method based on the derivation of closed-form Green's functions and the analytical evaluation of MoM matrix elements, the authors of [198] use the gradient search and GA in combination with electromagnetic simulation methods to evaluate the potential of using this approach as a CAD tool. In the work [199], the application of optimization methods to design ultra-wideband antennas for multifunctional wireless devices is discussed. The antennas investigated in this work are monopoles with flat all-metal radiating elements. In [200], the authors presented a procedure to design broadband microwave radiation absorbers including frequency-selective surface (FSS) screens embedded in a dielectric media, using GA with binary coding or the so-called microgenetic algorithm (MGA). Such a variant of GA simultaneously and optimally selects the material in each layer, the thickness of each layer, the periodicity of the FSS screen in the X and Y directions, its placement in the dielectric composite material, and the FSS screen material. The MGA was proposed by Goldberg [201]. This approach avoids premature convergence and can show faster convergence to the near-optimal region compared to the conventional GA for the multivariate multimodal problem [202]. In 2002, the authors of [203] developed a planar broadband antenna by combining GA and MoM. GA was used to optimize the shape of the metallization pattern in order to minimize the return loss in a selected frequency range, while MoM was used to estimate the return loss.

Further, the authors of [204] have successfully used the multistructure method of moments (MSMoM) presented in [93] to design printed antennas. The main advantage of MSMoM is that it provides a way to organize calculations, which is especially suitable for an iterative optimization procedure. In a classical electromagnetic simulation tool, the computation takes place within an optimization loop, so a new simulation procedure is required for each new structure. Thus, the simulation time increases if multiple structures need to be analyzed. By itself, MSMoM is not a fast solver since it does not reduce the computation time for a single simulation. The initial idea behind MSMoM is to simultaneously analyze multiple structures, parts of which are slightly different, with a single simulation. This concept avoids a large number of repetitive computations and therefore reduces the computation time for tuning the optimized structure considerably. MSMoM was implemented for two-dimensional structures and successfully applied to the classical patch antenna design [205], and then improved in [206] where the same results were obtained as with the classical MoM, but with a significant reduction in the time spent. Later, the same authors managed to significantly reduce the computation time compared to a direct MoM/GA implementation. This was achieved by excluding the EM simulation from the iterative optimization cycle [207].

3.6. Special Cases

As optical metasurfaces become more and more common, the fields of their application are becoming more and more complex. In [208], an adaptive genetic algorithm (AGA) was presented as a powerful method of evolutionary optimization capable of solving complex design problems and considering a large number of parameters in the field of optical metasurfaces. To demonstrate the advantages of the AGA method over traditional design methods, the authors solved several problems of interest to the optical community. For example, the binary plasmonic reflection matrix was successfully optimized for beam control applications. The authors showed that large datasets generated by GAs can be used as data for next-generation computational algorithms such as machine learning and deep learning. A new methodology was presented to directly design planar directional antennas [209]. Surface shaping was achieved by adding/removing hexagonal cells; this made the optimization efficient and free of manufacturing tolerances. The design is based on GA combined with an MoM-based electromagnetic solver. The proposed method opens the way for designing planar antennas with very high gain at high frequencies.

4. Reconfigurable and Smart Antennas

4.1. Overview

Fundamental limitations in the characteristics and bandwidth of compact antennas have been extensively investigated. For example, in [210], new limits, added to those found in [211], were presented for the minimum radiation achievable with an antenna whose conductor arrangement corresponds to a spherical surface. It is preferable to use a technique that allows several different radiators to share the same volume, and to enable the change of the antenna's performance characteristics to bypass such limits as operating resonance frequencies, polarizations, impedance bandwidths, and radiation patterns independently. This technique is called reconfigurability in antennas, and it may be implemented electrically using electronic switching components to redirect antenna surface currents, or optically using photoconductive switching elements, or even physically by changing the geometrical structural aspects of the antenna. Additionally, using the smart material technique, reconfigurable antennas can be implemented using such materials as ferrites and liquid crystals.

Tunable or reconfigurable antennas have received increasing attention, for example, in [212,213], where electronic switches such as PIN diodes were used, and in [214], where the field effect transistors (FET) were used. The authors of [215] exploited electromechanical switches such as relays and micro-electromechanical systems (MEMS) switches, and later the same in [216]. Electromechanical systems in which electrostatic actuation causes a global deformation of the radiator are also used to design reconfigurable antennas as in [217,218]. In addition, the design of such antennas can include silicon substrates in which plasma regions with high conductivity are created by the injection of a DC current [219].

In general, the most important task was to control the antenna's electromagnetic characteristics, but paying attention to the system-level performance is similarly important [220]. The reconfigurability itself provides degrees of freedom readily exploitable during the design process, so iterative EM analysis/geometry correction is possible in order to come up with an optimal geometry. This was one of the goals of the work [221]. The paper presents a novel patch antenna geometry that is well suited to the frequency reconfigurability achieved through the use of switches connecting metal portions to the ground plane. The optimization procedure combined a global optimizer (GA) with a local optimizer (1-bit neighborhood) for the geometry, and an exhaustive search for the switching patterns. During the optimization, the characteristics and antenna performance were evaluated using MoM simulation. The resulting structure had good characteristics, such as small size, space filling, and good return loss.

4.2. Reconfigurable and Smart Antennas: GA

Numerical optimization of antennas is increasingly used to achieve results in the case of strict requirements. GAs have been used to optimize reconfigurable antennas such as optimizing the far-field patterns of antenna networks [222] or passive microwave components [223]. Another area in which the use of GA designs shows promise is the development of "smart" antennas [224]. GAs have been successfully applied in automated antenna design problems with tight specifications and strong constraints, often resulting in structures whose topology and methods of operation are not intuitive [225,226]. The authors of [227] described an EM GA optimization (EGO) application developed for a cluster supercomputing platform. A representative patch antenna design example for commercial wireless applications was described in detail, which illustrates the versatility and applicability of the method. They showed that EGO allows combining the accuracy of full-wave EM analysis with the robustness of GA optimization and the speed of a parallel computing algorithm. The GAs have also been used to find the best states in reconfigurable antennas with many degrees of freedom [228]. The GA and MoM were also combined in a novel method used to design and optimize frequency reconfigurable pixel antennas in [229]. This method utilizes a multi-objective function that is efficiently computed by using only one full electromagnetic simulation in the entire GA optimization process. An attempt was also undertaken to minimize the number of switches in the design. The method was demonstrated by an antenna structure consisting of a rectangular grid of pixels adjacent to a ground plane and using radio frequency MEMS switches to achieve the reconfigurability. The results demonstrated that reconfigurable antennas can effectively be designed with a minimum number of switches using an efficient optimization method. Using momentum and hyperoptimization methods, the authors of [230] developed and optimized the radiation and absorption characteristics of modified Yagi-Uda (YU) nanoantenna arrays. Four antenna geometries were considered: a conventional YU powered from a voltage source and a transmission line, and a YU with a loop element powered from a voltage source and a transmission line. Numerical electromagnetic modeling of these nanoantennas was performed by the method of moments. The optimization method used was adaptive fuzzy GAPSO, which is based on the hybridization of a genetic algorithm (GA) and particle swarm optimization (PSO). The optimized results show that the modified YU nanoantennas have better gain, directivity, and radiation efficiency characteristics compared to conventional YU antennas.

4.3. Reconfigurable and Smart Antennas: The Field of Use

Using the MoM, a novel frequency reconfigurable H-shaped patch antenna was designed in [231]. The reconfigurability in frequency is obtained by connecting two switches in a basic antenna by using MEMS. By controlling the switches, the antenna can be operated at three different frequencies. The effects of antenna length, size, substrate, thickness, and shape have been evaluated and produced good and acceptable 2D radiation patterns in elevation plane, gain, and efficiency. The work [232] proposed a novel concept of a beam-reconfigurable antenna, which rotates the beam of a two-element Yagi antenna without mechanical rotation. The antenna was based on an arrangement of six wires extending radially from the apex of a support. A switch circuit was used to combine two wires into the driven dipole and two wires into the parasitic dipole. Using this concept, improvements are expected from the use of horizontal dipoles or folded dipoles and additional radiator types can be configured, such as single-dipole or vertical top-loaded radiator antennas. Realizations of the concept seem to be feasible up to the low microwave frequency range by suitable miniaturization of the switching circuit, and such reconfigurable antennas may provide diversity gain in communication systems. Later in [233], the authors presented a conventional single-band microstrip antenna in the shape of a pentagon. The 1×2 and 1×4 matrices of the conventional patch antenna were designed to increase its gain and directivity. The conventional patch antenna was converted into a dual-band antenna using the fractal geometry of the Dürer pentagon and varactor diodes. The proposed antenna can be used for satellite communications and radar applications. In the work [234], a square Koch fractal slot antenna for the UHF band is presented, which has FR4-G10 and Cuclad 250 substrates. The contribution of this work is in the use of fractal geometry to create an inexpensive slot antenna operating at UHF frequencies on a limited CubeSat surface area, and to optimize it for possible use with solar cells.

In the work [235], a printed annular reconfigurable metasurface (ARM) comprising concentric ring slots etched in a metal screen loaded by stubs and varactor diodes was proposed to realize two-dimensional leaky-wave antennas (LWAs) operating on a pair of azimuthally independent TM and TE cylindrical leaky modes. The ad hoc method of moments approach was used for the calculation of the antenna's far-field patterns. In the work [236], a novel design method of electronically controlled beam-reconfigurable antennas using a liquid crystal metasurface was proposed. The metasurface is a two-dimensional plane composed of a series of sub-wavelength liquid crystal unit cells. The beam in desired radiation direction can be achieved by varying the impedance distribution of the liquid crystal metasurface according to a microwave holographic technique. Such a beam-reconfigurable antenna with a low-profile and smoothly adjustable beam has great value in the practical applications of mobile communications. An antenna in [237] is based on the metamaterials concept and operates in the terahertz (THz) band for integrated applications. The proposed antenna consists of five layers of polyimide and aluminum as top and bottom substrates, a radiation section, a grounding layer, and a power line. To achieve high-operating parameters without increasing the physical dimensions of the antenna, the waveguide properties integrated into the metamaterial and substrate were applied to the antenna design. This was achieved by making linear tapered slots on the top surface and metal through-holes throughout the middle ground plane connecting the top and bottom substrates. The results confirmed that the proposed low-profile antenna, with a compact size, a wide bandwidth in the terahertz range, and an economical and easy-to-fabricate configuration, could be suitable for integrated circuits in the terahertz range. A new design of periodic metasurface grating was proposed in [238] to increase the bandwidth and gain while maintaining a low-profile antenna scheme. The results show that the proposed antenna may be potentially attractive for applications in satellite communications in the Ku-band. The five-band metamaterial absorber proposed in [239] consists of a three-layer structure of an upper metal resonator, an intermediate dielectric layer, and a lower metal plane. Such an absorber has potential applications in multiband electromagnetic invisibles, bionic sensors, and instruments for measuring thermal radiation.

The paper [240] presents the results of research and development of antenna complexes for MIMO systems based on the use of fractal radiators and chiral metamaterials. In particular, the paper considers complexes based on radiators in the form of Koch, Hilbert, or dipole Sierpinski fractal curves, and triangles with fractal partitions by medians. These antenna complexes with fractal radiators located on substrates of chiral metamaterials were considered for a 2×2 MIMO system, in which the same antennas are used for receiving and transmitting. The gain in spectral efficiency of the MIMO system with such antennas was evaluated in comparison with antenna complexes in which fractal radiators were located on dielectric substrates. The results of the study of the impedance and spatial characteristics of these antennas showed their performance and the possibility of implementing them for multifrequency (multiband) systems. It is shown that such antenna complexes allow for the realization of increased spectral efficiency of MIMO systems in several frequency bands. In [241], a closely spaced 2 × 2 dual-band MIMO antenna with high isolation based on a half-mode substrate integrated waveguide (HMSIW) is presented. Dual-band operation of the antenna element is achieved by loading the rectangular section outside the radiating aperture of the HMSIW resonator. This resonator is coaxially excited, whereas the rectangular section is excited via non-contact through the HMSIW aperture. Good isolation is achieved by using spatial and polarization separation. A decoupling gain of about 10 dB is achieved by inserting small neutralization lines (SNLs) in the center of the MIMO antenna between the antenna element radiators without increasing the size of the original MIMO antenna. With an edge-to-edge distance of only 0.036 λ0, the measured decoupling in both lanes was above 35 dB. In addition, the separation characteristics of the proposed MIMO antenna are suitable for a high-quality MIMO antenna system. In [242], a dipole antenna based on a decoupling bandpass filter (BPF) was developed. The BPF uses two U-shaped resonators mounted on the left side of an open transmission line and two L-shaped loops to generate signals with equal amplitude and inverse phase. In this way, the BPF volume is halved and the distance between the two output ports is considerably reduced. The BPF is then integrated with the dipole. Instead of a traditional L-shaped line with a wide ground plane, two thin microstrip lines of 1 mm width are used to connect the dipole and the BPF. The antenna bandwidth is further extended by merging the resonance of the dipole and the BPF. As a result, the new BPF and the dipole based on it have great potential for 5G MIMO applications. In [243], the researchers present a four-layer broadband directional antenna with symmetrical double polarization and an equilateral triangular slot, as well as a CPW power supply suitable for wireless laboratory measurements. The antenna structure consists of two equilateral triangular slots with a symmetrical pair of stubs. There are four layers consisting of two antennas and two reflectors; each antenna is connected to a CPW port, and these antennas are perpendicular to each other. A proper choice of the length and width of the slots gives a much broader bandwidth in impedance, dual polarization, and high decoupling. In the work [244], an automatic antenna design method based on the shape-blending algorithm was proposed. The algorithm was used to construct the shape of a wide slit antenna powered (fed) by a coplanar waveguide (CPW). First, two basic shapes were chosen as the initial and target shapes. Then, a shape-blending process was applied to obtain a series of shapes that were used as the geometric structure of the wide slit. In this way, a series of CPW-fed broadband antennas was obtained. These had similar but gradually changing characteristics. Experimental results have shown that the resonant frequencies change with the change of the slit shape in a certain frequency band, and the proposed antennas can be used in C-band and X-band radar applications. In the work [245], a new three-zone unit cell with electromagnetic bandgap (EBG) of compact size was designed, fabricated, and tested. The proposed EBG unit cell was based on a square mushroom-shaped EBG (M-EBG) structure with an interdigital coplanar waveguide (ICPW). By this method, the size of the proposed ICPW-EBG structure was reduced from λ/2 to λ/4, compared to the size of the standard M-EBG unit cell. The ICPW-EBG reflector can improve the directivity of the dipole antenna. The developed reflector ICPW-EBG with the dipole antenna provides operation in three bands, a low profile, and high gain which are appropriate for today's wireless communication systems.

In [246], three configurations of compact flat multi-loop antennas were proposed as candidates for the standard 5G frequency band. Each antenna consisted of the same feeder part, but with different designs of the dipole, director, and reflector parts. In [247], the classic Chinese window array structure was combined with a multiband microstrip antenna design that can be used in wireless mobile terminal equipment. The antenna radiator has a rectangular bending structure with four loops, which increases the effective current path of the antenna radiator in a limited space, so that the whole antenna becomes miniaturized. The branched phase set structure of four rings increases the current path of the antenna, making the antenna multiband. Test results show that such antennas can cover four bands and give out six main frequency points to cover different navigation systems. The application of antenna arrays in various designs and elements is also widely used to improve the design and performance of millimeter range antennas, to achieve the characteristics of high gain and fast-beam formation with the possibility of wide scanning [248]. In [249], for 5G applications, a compact substrate integrated waveguide (SIW) antenna array was proposed. This array consists of four SIWs fabricated on a single substrate layer. In each resonator there is a rhombic-shaped slot and a slot with a triangular split ring, which resonate on the TE101 and TE102 modes at 28 GHz and 38 GHz, respectively. Given these characteristics, including two bands, high gain, narrow beamwidth, miniaturization, and single layer, the proposed antenna array is a suitable candidate for 5G millimeter band communication systems with the flexibility to switch operating frequency bands based on channel-quality changes. In addition, a reconfigurable Vivaldi antenna array was designed using a power divider for 5G applications in [250]. The proposed antenna consists of an array of the Vivaldi antennas, which is fed through a power divider. The power divider splits the power equally into two branches of the Vivaldi antennas. By implanting a pin diode, the desired frequency bands are achieved.

Changing the antenna shape to meet the requirements was used in the work [251] to increase the bandwidth of a slot resonator antenna using a triangular slot on a half-mode substrate of an integrated antenna with a waveguide structure. This was influenced by the position, length, and width of the triangular slot. The corresponding combination of the slot position, length, and width can create a hybrid mode between the TE_{101} and TE_{102} modes, which depends on their phase. In [252], the authors proposed and developed two microwave systems with horizontal and vertical polarization to select the optimal configuration for pipeline imaging applications. First, a pipeline containing crude oil was modeled, and its thermal and dielectric properties were proposed. Then, the characteristics of the antenna array were optimized. Different numbers of antenna elements were successfully investigated in both vertical and horizontal polarization to find their optimal number for pipeline applications. These systems can be used as accurate sensors to manage fluid flow in pipelines during the production process in the oil and gas industry because they can display changes in phase ratio in real time. By contrast, other, less accurate methods including mechanical, optical, X-ray or gamma-ray, ultrasound, nuclear magnetic resonance (NMR), and, in rare cases, microwave methods cannot do this. In addition, a new type of frequency-reconfigurable antenna, the ferrite slab–loaded substrate integrated waveguide antenna, was proposed in [253]. Due to the loaded ferrite, the operating frequency of the antenna can be tuned in a wide frequency range by bias magnetic field and the position of ferrite slab in the antenna. The experimental results proved the frequency tunability of the antennas and showed that the tunable range could be much larger than that of electronic reconfigurable antennas. This type of antenna's radiation performance may be promising in wireless communication systems. A reconfigurable reflect-array antenna, which exploits the magnetic anisotropy of a biased ferrite substrate, was studied in [254]. The unit cell of such an antenna is composed of two centered metal square loops placed on the grounded ferrite substrate. The analysis of such a unit cell for extracting its scattering information was carried out by Galerkin's MoM. The results obtained in [255] showed that the use of a reconfigurable antenna and intelligent antenna selection strategy onboard a UAV provides a higher average signal-to-noise ratio compared to an omni-directional antenna in both line-of-sight (LOS) and non-LOS scenarios, and is more resilient to co-channel interference.

It is necessary to note that now, studies on reduced time of SLAE solution are mostly based on the effective use of computing resources of workstations (multicore, computer cache memory, graphics processor, etc.), whereas the hidden resources of algorithms that format and solve SLAEs themselves remain practically unexplored. When geometrical dimensions of a structure change, there are significant changes in the matrix elements, which makes the use of direct methods ineffective. Iterative methods also become inefficient without additional adjustments or recalculation of the preconditioning matrix.

4.4. An Example of the Current Problem

As we know, antennas are an important part of REDs, so it is important to improve their radioengineering and mass-size characteristics, which are especially relevant for on-board equipment. For example, Figure 1 shows spacecraft of the "retranslation" category [256]. Note that their mass decreases with each release, while their complexity increases. At the same time the antenna weight remains considerable (Figure 2) [257]. This is important because the cost of 1 kg cargo delivery to orbit can reach $40,000. Therefore, it is required to optimally design antennas.

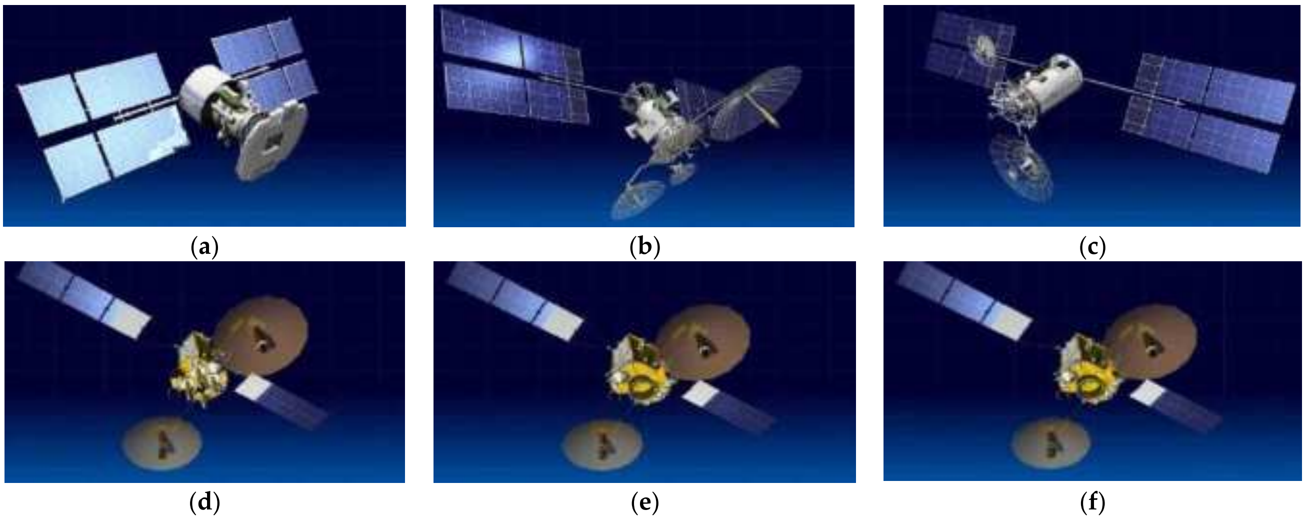


Figure 1. "Retranslation" category spacecrafts: Potok (2300 kg) 1982 (**a**); Luch (2400 kg) 1985 (**b**); Luch-2 (2420 kg) 1995 (**c**); Luch-5A (1150 kg) 2011 (**d**); Luch-5B (1350 kg) 2012 (**e**); Luch-5V (1150 kg) 2014 (**f**) [256].

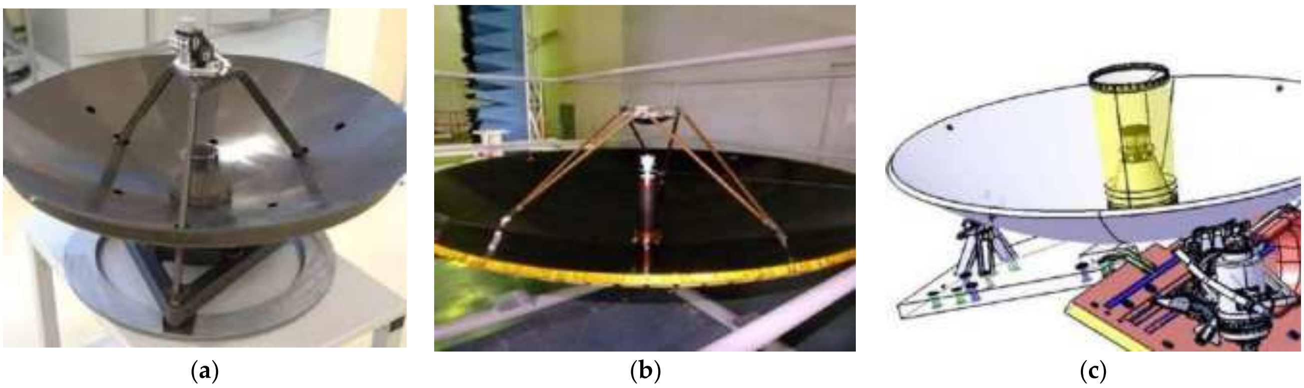


Figure 2. Examples of antennas: Precision antenna Ka-band (6.3 kg) (**a**); double-mirror parabolic Ka-band antenna (10.5 kg) (**b**); broadband antenna-feeder device (55 kg) (**c**) [257].

To solve this problem, engineers use composite materials (Figure 3) [258] and transition to a grid of wires (Figure 4) [259]. For example, at the Institute of Computer Technologies, one of their projects studied the properties of orbital and ground antennas from composite materials working in the bands of 18.2–21.2 GHz and 42.5–45.5 GHz. Antennas of such frequency ranges should possess simultaneously high geometrical stability, stability to temperature impacts, raised rigidity, and small mass. Computer simulation played a very important role at every stage in this project and allowed for parametric optimization of antenna designs. Modeling and optimization of these types of antennas, including those composed of thin meshes, remains a non-trivial task. In this case, the best simulation results can be achieved only using electrodynamic modeling in the range of a large number of parameters, which is often hampered by high computational cost requirements. Therefore, it is important to identify and study new approaches to improving the efficiency of such antenna modeling.

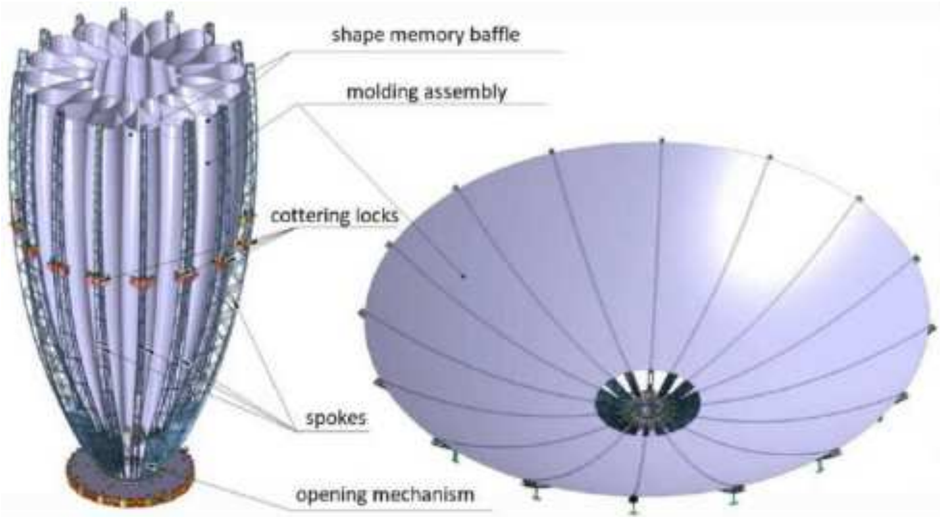


Figure 3. Example of an antenna made of composite materials [258].

Figure 4. Example of an antenna in the form of a grid of wires as a transformable antenna with gilded wire mesh (mounted on Luch 5A, Luch 5B, and Luch 5V) [259].

5. Wire-Grid Sparse Antennas

5.1. Overview

The history of computer development goes back more than 60 years. Over the years, technology has improved rapidly, resulting in the evolution of computers from large mainframes of collective use to today's superpowered workstations. During this time, computer technology has become a tool for many people. Therefore, the development of computer technology explains the intensive work undertaken to create new methods in all branches of science, focused on the computer. Modern CAD involves the use of computers, and they develop in the direction of comprehensive automation in all stages of the product life cycle. This situation also emphasizes the problem of creating new methods of design and construction, and mathematical and software algorithms for solving various scientific and engineering problems [260]. Thus, for example, applied electrodynamics includes the tasks of designing antennas and antenna arrays, synthesizing planar multilayer absorbers and multilayer dielectric structures, developing frequency-selective surfaces and various electromagnetic devices, etc.

To solve electromagnetic field problems, there are various approaches that are reduced to analytical or numerical solutions of the corresponding integral or differential equations in the frequency or time domain. Analytical solutions are accurate, however they are obtained only for a limited range of simple structures. Thus, the use of a thin-wire approximation allows us to analyze various antennas [261]. In this case, such an approximation can be extended to the wire grid, for example, to design lens antennas [262–269]. Microstrip [270,271], reflective [272,273], and indoor antennas for mobile communications [274] can also be represented in the form of a wire grid. In addition, the thin-wire approximation is applicable to solving scattering problems. Thus, for example, the work [275] presents an analysis of the reflection transmission of a plane wave at oblique incidence on a wire grid parallel to the plane interface of two homogeneous dielectrics. In this analysis, the space on both sides of the interface is represented in the form of transmission lines.

The use of numerical methods allows for the analysis of antennas of arbitrary configurations [276]. Application of these methods allows moving beyond continuous functions to their discrete analogues, thus making a problem solvable. As a result, the problem is reduced to finding an approximate solution of Maxwell's equations. A distinctive feature of numerical methods is the partitioning (discretizing) of the geometric model into small cells (grid construction) [277]. With this feature, both partial differential equations (DE) and integral equations (IE) can be solved in the time domain. PDEs can be solved using the transmission line matrix method (TLM) and the FDTD as well as the finite element time domain (FEMTD). IEs, in turn, can be solved using the partial-element equivalent circuit method (PEEC). In FD, the solutions can be either in the high-frequency (HF) or low-frequency (LF) domain. In LF, the equations can be solved with DE and IE. The former can be solved using the FEM, while the latter can be solved using MoM and PEEC. To find a solution in the HF domain, asymptotic methods based on physical and geometrical optics as well as diffraction theory are used.

The peculiarities and the results of MoM application in the analysis of wire-grid objects are presented in many works as the solution of scattering tasks and antenna tasks. Thus, the use of the collocation method (a special case of MoM) to solve the scattering problem by wire objects is considered in [278]. To increase the accuracy of calculations, additional boundary conditions are imposed on the segments of each wire. In this case, the required computational cost significantly decreases, and the obtained results are in good agreement with the experimental ones [279]. In [280] it is shown that when exciting a parallel wire grid located near a flat interface between two homogeneous media with an electromagnetic wave, polarized so that the magnetic field vector is perpendicular to the gird, it can be represented by a shunt element in an equivalent transmission line circuit. In the work [281], the researchers investigated the application of the iterative method to solve a SLAE obtained by representing a spherical antenna by a grid of wires. They also established the optimal parameters of the method, which give close results to those obtained theoretically. The work [282] presents an effective computational method for obtaining RCS of an electrically small aircraft, the fuselage of which is described by a grid of wires. It is shown that with the correct choice of the wire segments' radii, the simulation and experimental results are close for the polarization of the incident wave parallel to the fuselage axis. In the case when the polarization is perpendicular, the agreement of results is worse. In [283], a method for estimating the scattering of a rectangular wire grid, based on the use of CG and fast Fourier transform methods, is proposed. In addition, the principle of equivalent radius is used to determine the reflection coefficient. In [284], an example of the scattering problem for an infinite circular cylinder is used to analyze the influence of the wire diameter of the wires on the modeling accuracy. It is shown that the best accuracy is achieved when the wire satisfies the empirical rule of "equal surface area". At the same time, it is revealed that wires that are too thick are just as harmful as wires that are too thin. In addition, it is shown that the observance of boundary conditions between the wires is not a reliable check on the validity of the simulation results.

Regarding antenna tasks, the authors in [285] present the results of the development of a wire-grid-antenna modeling system, and in [286] the NEC system, which is used to estimate characteristics such as average power gain and electric near and far fields of a monopole antenna mounted on a cubic base over a perfectly conducting ground plane. It is shown that the simulation results are in good agreement with the experimental data and the results of another independent electromagnetic modeling code. In [287], a new methodology is proposed to calculate scattering characteristics of three-dimensional conducting bodies of an arbitrary shape, and the features of the modeling system that implements it are described. In the work [288], the peculiarities of the development of a graphical user interface of a GEMACS antenna modeling system are presented. In the work [289], the authors considered a number of problems arising that simulate complex three-dimensional configurations, such as the antenna-airplane system, where the fuselage of the aircraft is approximated by a grid of wires. It is shown that, despite some limitations, the results can be obtained with acceptable accuracy. In [290] an algorithm for local mesh partitioning is considered that takes into account the discontinuous nature of the interface between large and small cells for a smooth transition of strongly varying field components. In the paper [291] the convergence of the wire-grid model is evaluated when the number of grid cells changes. This allowed the researchers to obtain correlations between the minimum grid size and the required modeling error. Ultimately, the researchers determined the sensitivity of the original antenna design to small changes in its shape.

After creating RWG functions [292] that are used to describe surfaces by a set of triangles, the application of wire grids has become less popular. However, the comparison of the results of these two approaches often demonstrates excellent suitability of the wire grid [293] further, in solving practical problems, the computational cost can be significantly reduced with good simulation accuracy even compared to the experimental data [294,295].

One of the most important advantages of using a wire grid is the possibility of obtaining results when analyzing the scattering of large objects (e.g., ships, aircraft). Thus, the analysis of how the quality of wire-grid construction influences the results of modeling such objects was performed in the paper [296]. The general principles and conditions were established, and recommendations on applicability of such meshes were given. Their implementation allows for building a better grid and takes into account the features of the structure to be analyzed, while obtaining correct simulation results including the use of nonlinear wires [297]. The use of wire grids is not limited to conductive objects. Thus, they were considered for obtaining a scattered field by lossy dielectric objects [298] and anisotropic layer structures [299]. The "weak point" in the use of wire grids is the modeling of near fields [300]. In this case, as shown in [17], the check of the solution accuracy based on the coincidence of the wire boundaries is not always correct.

5.2. OCGA Method: Explanation

The method proposed here is named the Optimal Current Grid Approximation (OCGA) and is aimed at improving the development of radiators and scatterers of the electromagnetic field through the optimal approximation of the wire grid. Unlike well-known solutions, this can be achieved not by complicating but by simplifying the structures and by modeling without doing too much. This section outlines how this is achieved. This method can be used for manufacturing a wire-grid antenna. The main idea is to avoid using those wires with low currents in the antenna grid, since their contribution to radiation is small. This can be achieved by excluding from the grid those wires in which the current modulus is less than a given threshold. This in turn will make it possible to reduce the mass, windage, and dimensions of the antenna with controlled accuracy of its characteristics. This method can also be used for accelerated modeling of the electromagnetic field of a conducting surface with a sparse wire grid. The main idea is to avoid using wires with low currents in each simulation of the electromagnetic field of the wire-grid structure in a range of parameters, since their contribution to the total radiation is small. This can be achieved by adding a number of actions to the regular modeling process to identify these wires after the first modeling of the original (full) grid, and for any further simulation, only the resulting sparse grid with a smaller number of wires in a given range of parameters should be used. This will require less memory and time spent on the modeling, with controlled accuracy. Such an approach can be applied to obtain fast estimates of antenna characteristics. As a result, the same idea of identifying and eliminating essentially unnecessary wires can be used both in developing and modeling antennas and scatterers.

The proposed method is based on MoM and consists of the following steps:

1. Obtain an integral equation for a given structure from Maxwell's equations;
2. Describe (input or import) the task geometry;
3. Specify the required frequency range;
4. Select the required characteristics to be calculated;
5. Select the excitation source;
6. Construct the grid (divide the boundaries of the structure into N subdomains, select the type of basis functions, and approximate the desired function in each subdomain by the corresponding basis function);
7. Select the type of test functions and test the approximated desired function by MoM;
8. Calculate the entries of impedance matrix $\mathbf{Z}$ of order N and the entries of the SLAE right-hand side voltage matrix $\mathbf{V}$;
9. Solve the obtained SLAE $\mathbf{ZI} = \mathbf{V}$ to find the matrix of surface currents $\mathbf{I}$;
10. Calculate the required characteristics of the antenna from matrix $\mathbf{I}$.

What follows is the explanation and the algorithm of the proposed OCGA. This method is proposed as a method of modeling based on MoM, as mentioned above. The OCGA algorithm consists of the following. It begins with the construction of a geometric model of a real surface in the form of a wire grid. After that, an excitation with its parameters is set. Then, the wire-grid segmentation is performed. After calculating the entries of matrix $\mathbf{Z}$ that characterize the grid of the structure and the entries of matrix $\mathbf{V}$ that characterize the excitation, the SLAE $\mathbf{ZI} = \mathbf{V}$ can be solved. From the calculated elements of the matrix of currents $\mathbf{I}$ in the grid segments, the required antenna characteristics are calculated. Further, the previous steps are repeated when the parameters change in a given range. However, according to OCGA, before repeating the previous steps, the current vector element modules should be normalized with respect to their maximum. The choice of the normalizing criterium depends on the structure. For example, this step might be performed without taking into account the current in the excitation source, or the current matrix elements might be normalized with respect to the average current value. Then, the numbers of segments in which the current module is less than the given value of the threshold parameter are determined. The value of this parameter is set by the user or according to the manufacturer requirements or the desired accuracy). Later it will be called the grid element elimination tolerance (GEET). After that, the SLAE columns and rows that correspond to these numbers are cut out, and the remaining ones are shifted, which reduces its order. Finally, if necessary, a new "sparse" antenna, without the deleted elements that correspond to the eliminated SLAE columns and rows, is displayed.

5.3. Wire Grid: Application and Example

5.3.1. Real Commercial Parabolic Antenna

In order to demonstrate the applicability of OCGA, a dual-polarized parabolic commercial antenna [301] (Figure 5) is used. This antenna is designed for directional links with MIMO mode at the frequency band of 5 GHz. The electrical and mechanical parameters of this antenna are presented in Tables 1 and 2, respectively [301]. The measured results of the antenna gain are presented in Figure 6 [301].

(**a**) (**b**)

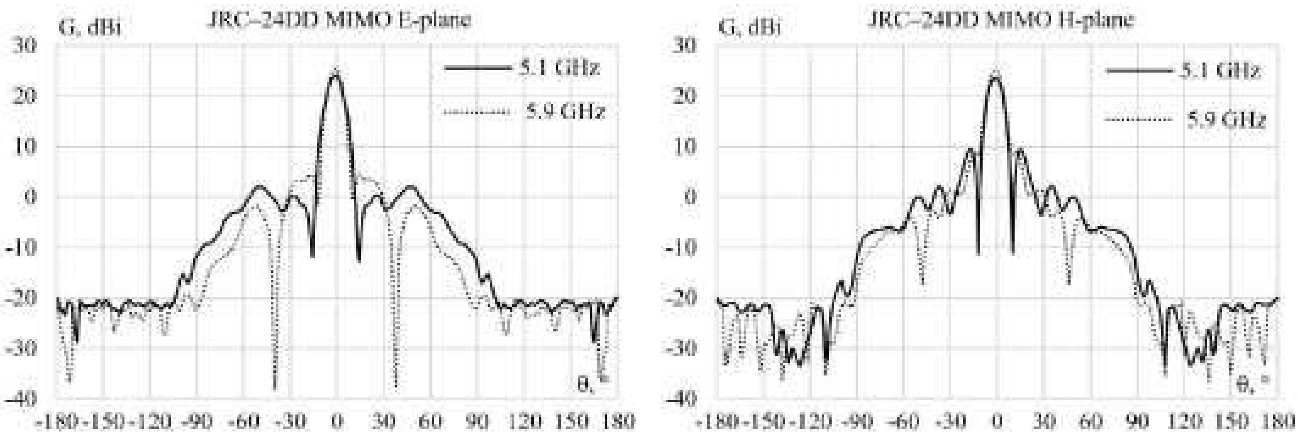
Figure 5. Actual image of the JRC-24DD MIMO antenna: front side (**a**); back side of the antenna (**b**) [301].

Table 1. Electrical parameters of the antenna [301].

Frequency Range	Gain	$VSWR_{5.1–5.9\ GHz}$	$Beamwidth_{-3\ dB}$
4.9–6.4 GHz	24.5 ± 1 dBi	≤1.4	9.0°

Table 2. Mechanical parameters of the antenna [301].

Parabola	Installation For Mast	Weight of Antenna
Ø 400 mm, Alluminum alloy	Ø 27–74 mm	2.3 kg (5.1 lbs.)

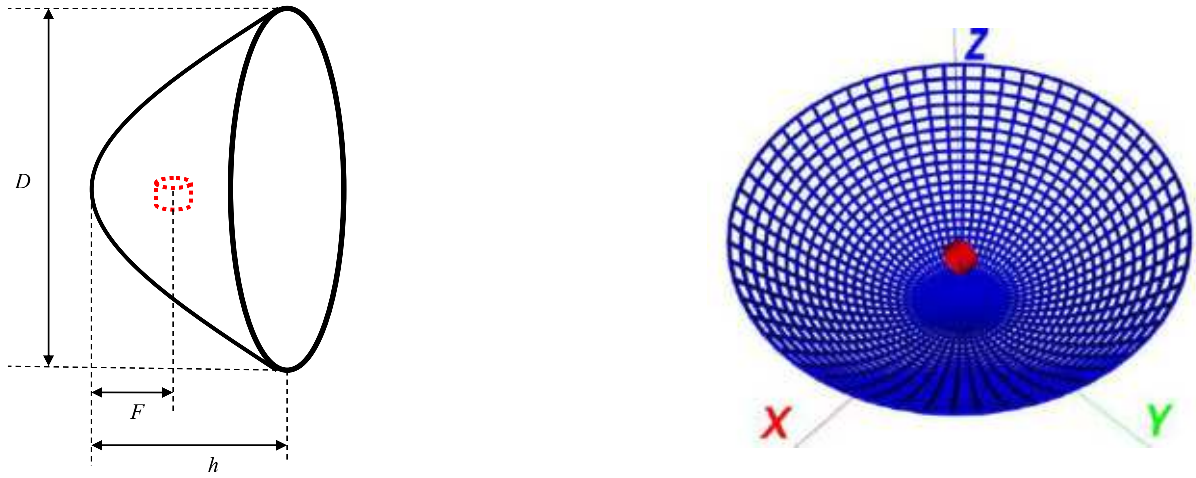


(**a**) (**b**)

Figure 6. Measured antenna gain of the JRC-24DD MIMO: E-plane (**a**) and H-plane (**b**) at 5.1 GHz and 5.9 GHz [301].

5.3.2. Modeling Using Wire Grids

The structure of the parabolic antenna is modeled using a MoM wire grid with the thin-wire approximation [302]. The isometric view of the parabolic antenna structure is presented in Figure 7, and its geometrical parameters are as follows: the diameter D = 400 mm, the depth h = 143.6 mm, the focus F = 75 mm. As an excitation source, a dipole is located at a distance equal to the focus of the reflector from the center of the grid, and its length is chosen equal to $l = \lambda/3.6$. The dipole radius is chosen equal to $R_1 = \lambda/3.6$ and the radius of grid wires $R_2 = \lambda/30$. The wire grid is built using 60 radial wires and 25 horizontal ones (Figure 7). According to the antenna datasheet [301], the simulation using the wire grid is performed at frequencies of 5.1 GHz and 5.9 GHz. The antenna gain is calculated in the E-plane and H-plane, and the results are compared to the measured ones and presented in Figure 8.



(**a**) (**b**)

Figure 7. Isometric view of the parabolic antenna structure (**a**); and the wire-grid modeled structure of the parabolic modeled antenna (**b**).

5.4. OCGA Method

5.4.1. General OCGA

Using the general OCGA method described above, the sparse antenna was obtained for the modeled antenna from [301]. Two different sparse antennas were obtained only to demonstrate the difference between the resulting sparse structure at each frequency. The current matrix elements were normalized with respect to the maximum current magnitude in structure wires. These sparse structures were obtained taking into account the GEET value equal to 10%. All the elements in which the current value was smaller than 10% of the maximum one were deleted. The resulting structures are presented in Figure 9. The antenna gain calculated using the wire grid (full structure) and those obtained using the general OCGA (sparse structure) are compared and presented in Figure 9 at 5.1 GHz and 5.9 GHz. The results obtained using the sparse antenna comparing to the measured ones and those obtained using the wire grid are consistent. The differences between them can be controlled by the GEET value. Moreover, the difference might arise with the frequency increase as shown in Figure 9. By comparing the full wire-grid structure and the sparse one, we found that the antenna mass reduced by 1.51 times at 5.1 GHz and by 1.42 times at 5.9 GHz. Meanwhile, the computational time spent on antenna modeling also reduced by 3.44 times at 5.1 GHz and by 2.88 times at 5.9 GHz. The required memory also reduced by 2.28 times at 5.1 GHz and by 2.02 times at 5.9 GHz.

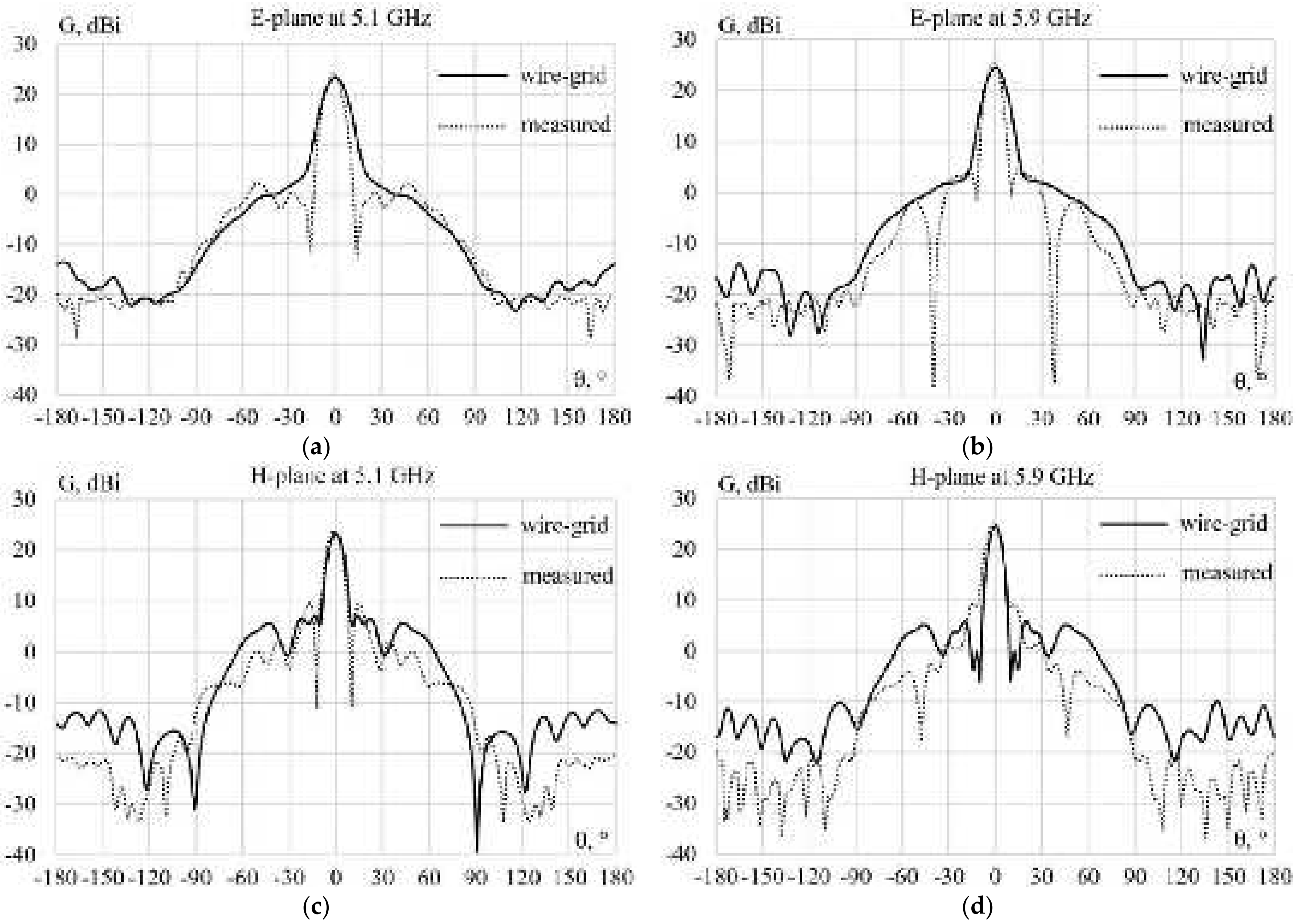


Figure 8. Antenna gain obtained using the wire grid comparing to those measured in [301] in E-plane at 5.1 GHz (**a**) and 5.9 GHz (**b**); in H-plane at 5.1 GHz (**c**) and 5.9 GHz (**d**).

5.4.2. Connected OCGA Approach

The structure of the sparse antenna obtained using OCGA might add technical difficulty in manufacturing such antennas, which might differ according to the structure. For example, in the case of sparse printed antennas using OCGA, the antenna's physical geometry can be manufactured easily, but for reflector antennas, as shown in Figure 9, using OCGA complicates their manufacturing. Since OCGA can be easily optimized and adapted, any condition might be applied depending on the requirements. To demonstrate this capabilities of OCGA, we proposed another approach of obtaining sparse antennas using OCGA. We assumed that the above sparse antenna needs to be manufactured and in order to construct the structure prototype, for example using a 3D printer before metallization, we modified the algorithm of OCGA to give a structure of physically connected wires. In other words, the wires are used only where the current flows. The sparse antennas obtained using this approach and their gain results are compared with those obtained using the wire grid and presented in Figure 10. The results show good consistency, and the structures seem to be easier to manufacture.

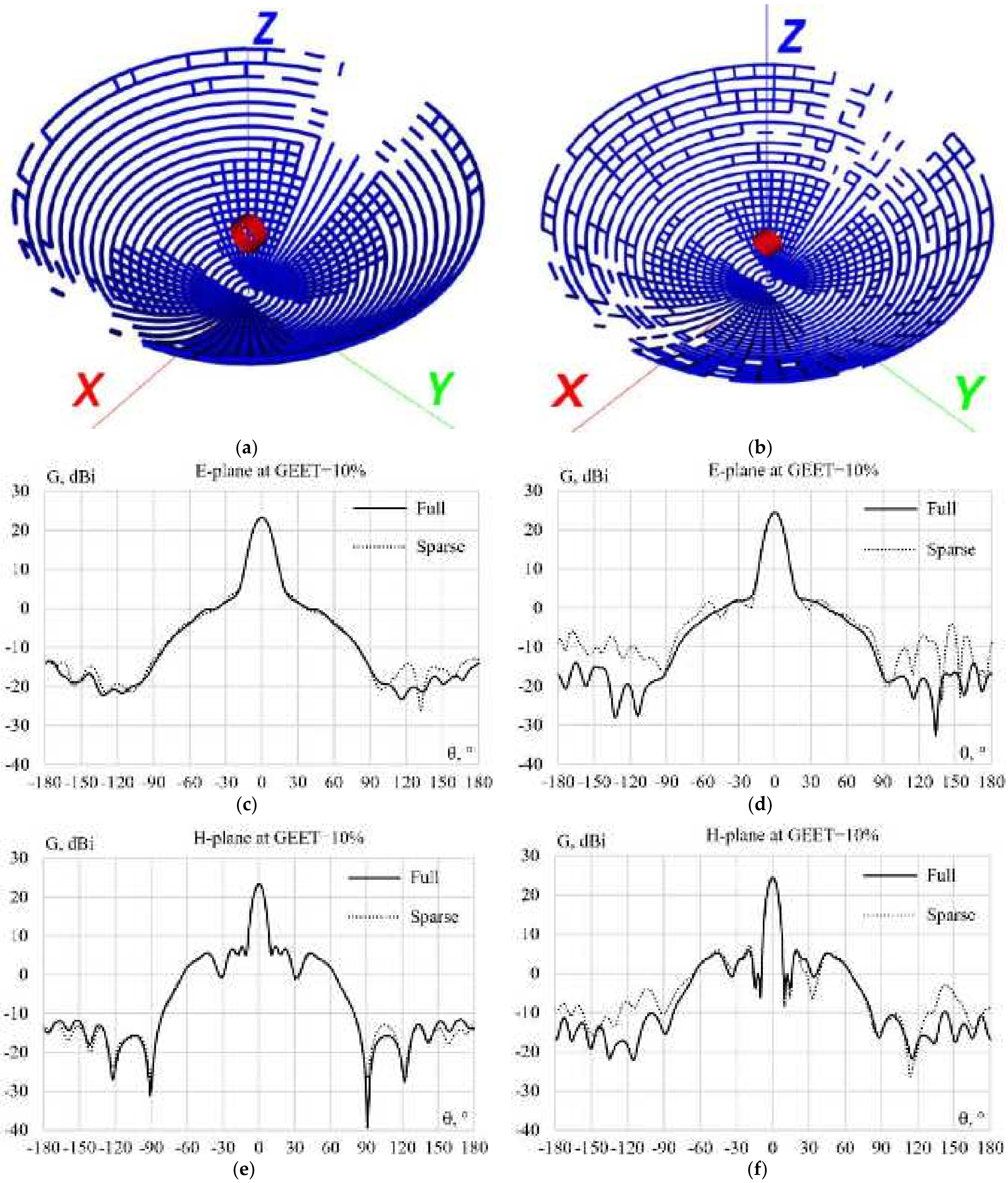


Figure 9. Sparse antenna structures obtained using the general OCGA (sparse) at 5.1 GHz (**a**) and 5.9 GHz (**b**) with GEET = 10%; and their gain results compared to those obtained using the wire grid in E-plane at 5.1 GHz (**c**) and 5.9 GHz (**d**); and in H-plane at 5.1 GHz (**e**) and 5.9 GHz (**f**).

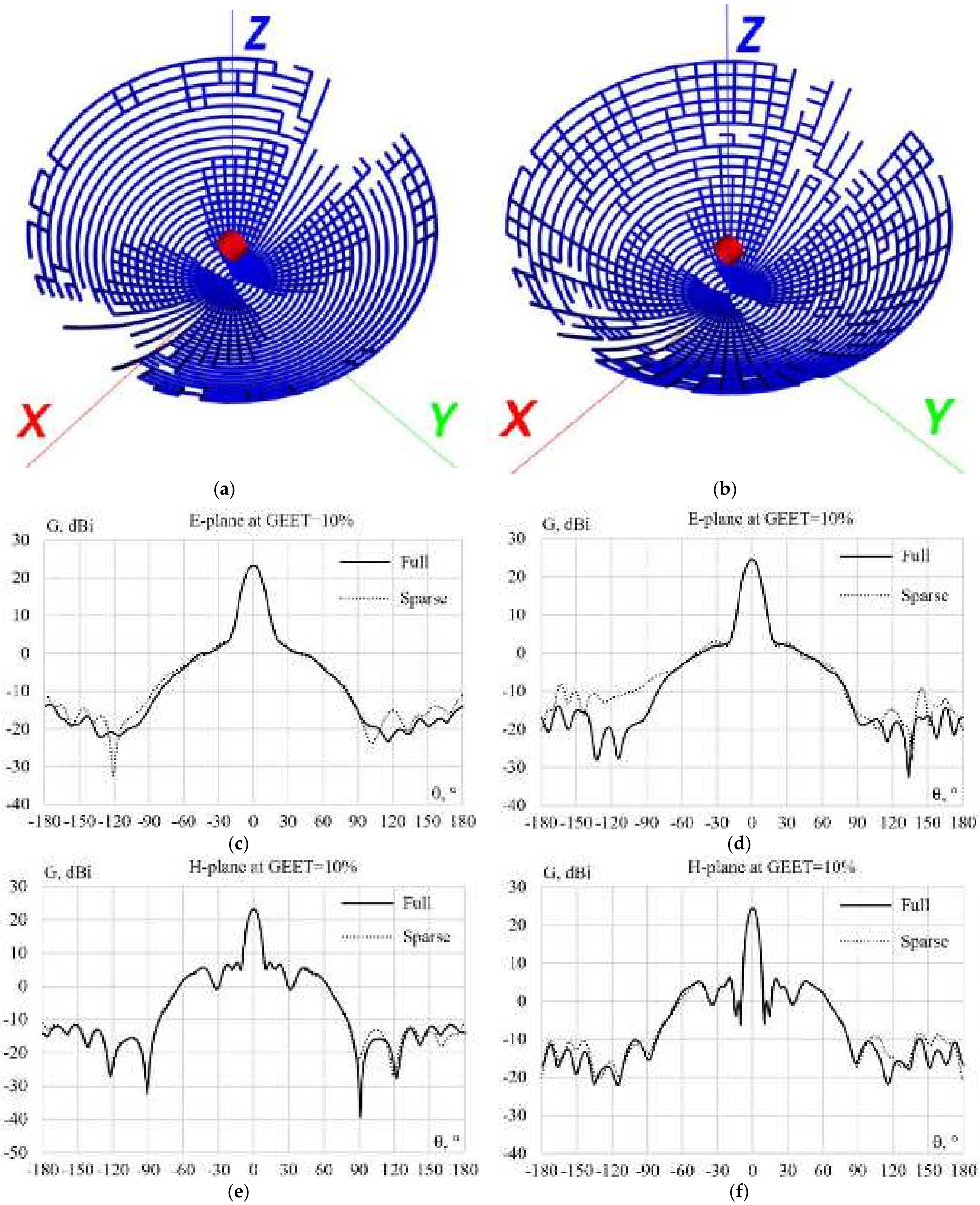


Figure 10. Sparse antenna structures obtained using the connected OCGA (sparse) at 5.1 GHz (**a**) and 5.9 GHz (**b**) with GEET = 10%; and their gain results compared to those obtained using the wire grid in E-plane at 5.1 GHz (**c**) and 5.9 GHz (**d**); and in H-plane at 5.1 GHz (**e**) and 5.9 GHz (**f**).

5.4.3. OCGA Sparse Antenna

In order to demonstrate the possibility of using sparse antennas obtained by the OCGA method, we took the sparse antenna obtained at a frequency of 5.9 GHz and modeled it at a frequency of 5.1 GHz. Then, we compared the results and presented them in Figure 11. As can be seen, using the OCGA method to obtain a sparse antenna from an antenna that operates in a frequency range does not significantly affect the characteristics of the new sparse antenna, considering the fact that the surface current flows in a different direction at each frequency.

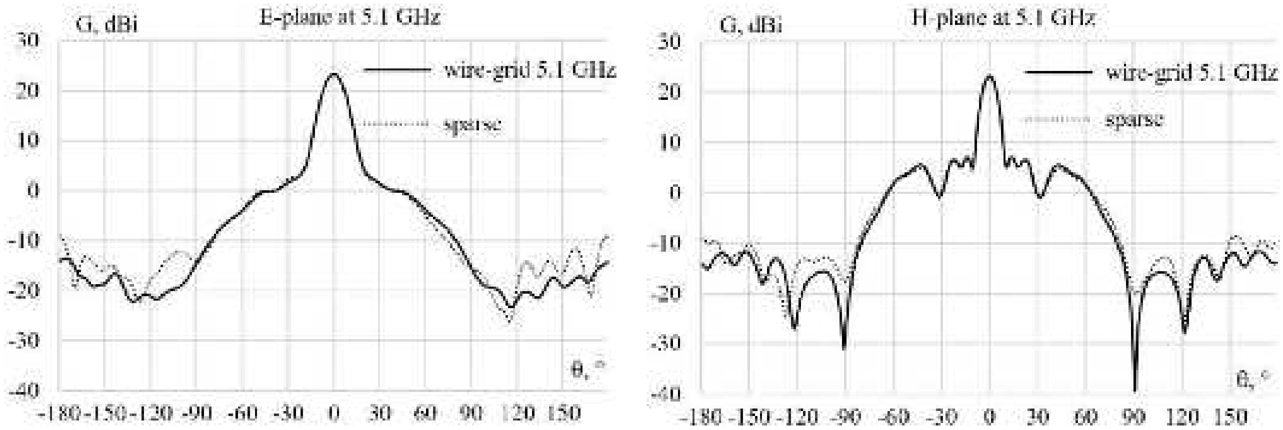


Figure 11. Antenna gain at 5.1 GHz obtained using the sparse antenna from the wire-grid structure at 5.9 GHz compared to that obtained using the wire grid (full structure) at 5.1 GHz in E-plane (**a**) and in H-plane (**b**) with GEET = 10%.

5.4.4. Discussion and Future Work

The OCGA method is under research and further development. This can be explained from the fact that the rectangular grid is universal for any excitations and frequencies, but is far from optimal (for example, by the criterion of reducing the mass and by the criterion of reducing the computational cost of modeling). Taking into account the arbitrariness of the coordinates of the beginning and end of any wire segment, the optimal grid should repeat the directions of the current on the surface and thickening at the places where the changes in the current are sharp. When modeling, this will reduce the order of the matrix of the SLAE that describes the structure (when modeling using MoM). But modeling the surface with a wire grid, according to the flow of currents over the surface, enables manufacturing the object itself not in the form of a real flat surface but in the form of a wire grid. Indeed, there is no need to use, for example, a sheet of solid metal if currents do not flow through it everywhere; it is necessary to leave the metal only where significant currents flow and remove it where insignificant currents flow. Then, for example, it is advisable to switch: from a wire grid to radial conductors; from a grid of strips with the same width to strips with varying widths in accordance with the current density; or from a solid volume to a certain structure with the combined use of the two previous options. Thus, this idea requires more investigation and further development.

Till now, this method has been applied to simple wire structures (the entries of the SLAE matrix can be calculated even analytically) using the thin-wire approximation (impulse basis functions and Dirac test functions, according to the classical work by Harrington). It is supposed to be used later, even though it requires a correct choice of the segment length at a given wire radius relative to the wavelength.

Further study should consider the possibility of reducing the number of antenna elements with a controllable change of their critical characteristics with a given GEET. This is appropriate to reduce the antenna's mass, size, sailing, required memory, and computer time consumption in a multivariate analysis or optimization of the structure on a large number of parameters.

In addition, it is crucial to investigate another important aspect: the applicability of the proposed approach. Thus, due to the non-use of unnecessary elements, it seems that this is effective for simple requirements (for example, radiation of the antenna at only one frequency). However, in the case of complex requirements (for example, radiation at several frequencies), the efficiency of this method will reduce. Meanwhile, this conclusion is ambiguous: despite the fact that the antenna currents flow differently at different frequencies, a preliminary study of this method showed the ability of the antenna to work in a fairly wide frequency range, such as 6–9 GHz. Moreover, the amplitudes and directions of the antenna currents changed insignificantly. As a result, the received "sparse" antenna (at a certain GEET) could easily replace the initial one (full grid). Therefore, it is expedient to investigate this question.

Moreover, the possibility and the efficiency of the transition from the orthogonal to the radial wire grid should also be considered, as it will allow the reduction of the redundancy of the orthogonal grid, by reducing by two times the number of segments (by replacing two cathetuses by hypotenuse), and possibly even more, by reducing the number of segments in the polyline (up to the radial line from the source to the structure edge). An illustrative example is the approximation of a biconical antenna by radial conductors only, without transverse ones, because no currents flow in them due to the symmetry.

On other hand, the OCGA itself can be modified as an optimization method to obtain the desired characteristics of the required antenna. Finally, it is possible to hybridize the OCGA with GA to optimize the structure designs, but this will be at the expense of computational costs. In addition, the analysis of the characteristic mode [303] of the antennas obtained using OCGA might help to understand the antenna's working principle, and to select its excitation location in order to obtain an optimal antenna structure, which should be considered in future studies of the OCGA method.

6. Conclusions

In this paper, we have undertaken an intensive review. At first, we concentrated on MoM and reviewed the studies of other researchers related to MoM and even to other computational methods. We tried to reflect the reasons why MoM investigations are still relevant. We also provided a considerable amount of research related to the use of MoM, as well as various methods used to accelerate the modeling and calculating processes. In addition, we mentioned some of the methods that are used when MoM is combined with other methods. Furthermore, some research in the review contained remarkable examples of using MoM for modeling antennas. Various optimization techniques have also been considered in this review as they are among the most important aspects in modeling antennas. Among these techniques, we considered genetic algorithms (GA) and the studies that employ the combination of MoM and GA in modeling antennas. In addition, we provided a review of reconfigurable and smart antennas followed by wire-grid sparse antennas. We also presented a new method that can be used in both development and modeling of antennas and scatterers, and demonstrated its capabilities.

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