Algorithm of nonequidistant segmentation of boundaries of conductors and dielectrics for computer-aided design of strip structures

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Abstract— **Algorithm for non-uniform segmentation of the boundaries of a system of conductors and dielectrics to improve the computer-aided design of strip line structures that are widely used in radioelectronic equipment for various purposes is proposed. The algorithm is based on the adaptive iterative choice of optimal segmentation and the previous iteration information about the surface charge density at the boundaries of the analyzed system. Features of algorithm implementation are described. Several variants of its implementation are considered. On the example of calculating a capacitive matrix of three configurations, the operation of proposed algorithm and of algorithms using uniform segmentation is compared. The effectiveness of the proposed algorithm from the point of view of minimizing the computing costs is shown.**

Keywords*— strip line; computer-aided design; adaptive iterative choosing of optimal segmentation; quasistatic analysis.*

I. INTRODUCTION

Strip line structures are widely used in radioelectronic equipment for various purposes as separate elements, printed circuit boards or filters for example, also as whole modules, phase shift module or antenna array for example. For computer-aided design of strip line structures a mathematical modeling becomes more and more important. It in general consists of the following steps: problem statement; equations formulation; model discretization (segmentation) and system of linear equations building; the system solution; results processing. Because of specifics of strip line structures the quasistatic approach is widely used for analysis of these structures. Solutions of one variant and many variant analysis problems are needed for design of strip line structures. These solutions can have high computational costs. Therefore, new algorithms development is relevant to decrease computational costs.

When making analysis the segmentation influences all the next steps. Segmentation type depends on a numerical method used for solving a problem. Triangulation, line division by segments, plane segmentation using rectangles, parallelepiped segmentation and also other segmentations can be used. A segmentation can be equidistant or non-equidistant. It can be performed iteratively when new segmentation is created on each iteration. In calculation error estimations for exit condition various parameters are used, such as: charge or current, charge density or current density, electric field or magnetic field and so on [1], [2].

When using the method of moments the quasistatic analysis of strip line structures is reduced to finding the electrostatic and electromagnetic induction coefficient matrices using segmented conductor and dielectric boundaries of an analysed structure. For computation of these matrices it is needed to solve an equation

$$\mathbf{S\Sigma=V} \tag{1}$$

where **S** – dense and square matrix of order N, **V** – matrix with dimensions $N\times N_{\mathrm{COND}}$ that consists of specified potentials on structure segments (1 V – on conductor-dielectric segments and 0 V on dielectric-dielectric segments), **Σ** – unknown matrix with dimensions $N\times N_{\mathrm{COND}}$, that is a charge density on the segments, N_{COND} – total number of conductors in the structure. Order of the matrix **S** is a sum of conductor-dielectric segments number (N_c) and dielectric-dielectric segments number (N_d). Entries of matrix **S** are calculated using information about parameters of segments. Structure of matrix **S** is schematically shown on Fig. 1*a*. If N_c^l is the number of segments at boundaries of l-th conductor, then $N_c = N_c^1 + \ldots + N_c^l + \ldots + N_c^{NCOND}$.

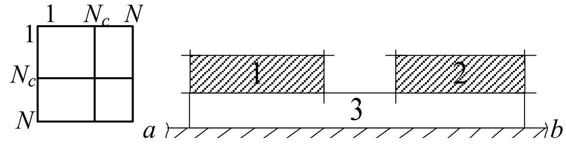

Fig. 1. Structure of matrix **S** (*a*). Cross section of coupled microstrip line (*b*). 1, 2 – conductors, 3 – dielectric substrate

Be specific, we consider cross section of coupled microstrip line (Fig. 1*b*). In this case we have N_{COND}=2, N_c=8, N_d=3.

In article [3] approach and algorithm for adaptive iterative choose of segmentation (AICOS) are described. AICOS algorithm are based on refining the equidistant segmentation at each iteration of the algorithm when solving electrostatic

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problems. Effectiveness of this algorithm was shown. Also it was noted that it is convenient to use information obtained at previous iteration to get smoother segmentation using local refining the segmentation (non-equidistant segmentation).

The aim of this paper is to present results of development of the non equidistant AICOS algorithm for analyzing the strip line structures.

II. ALGORITHM OF NON-EQUIDISTANT SEGMENTATION

To make next statements clear, the relation between steps of mathematical modeling using iterative segmentation refinement is shown in form of algorithm on Fig. 2.

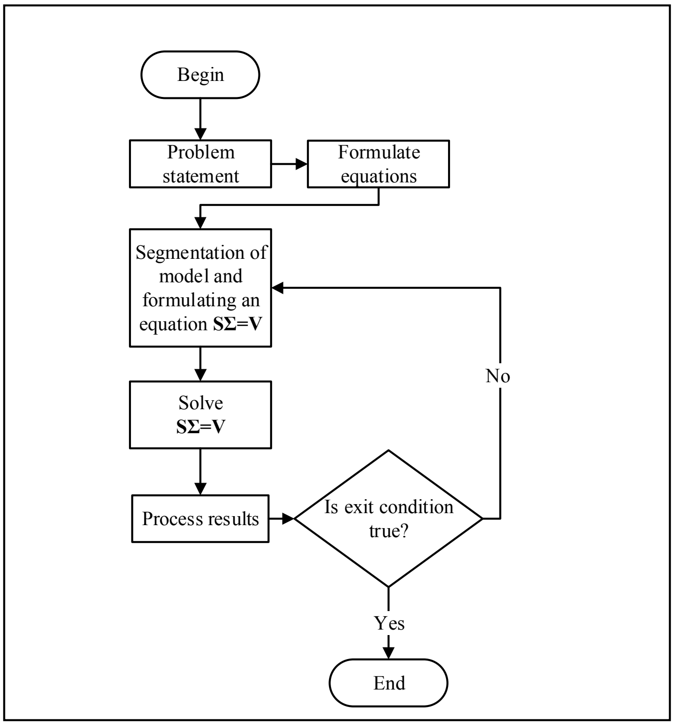

Fig. 2. Algorithm of creating a mathematical model using iterative segmentation increase

Using the algorithm from Fig. 2 we developed non equidistant AICOS algorithm:

1. Assign the modeling parameters (structure parameters, *tol* – precision of a controlled variable, ε – segmentation precision, *n* – number of divisions of a segment).
2. Let $j=0$ (j – iteration index, $j=0, 1, \ldots$), create a equidistant segmentation (for example, each boundary is divided into *n* segments), solve the equation (1) and use the solution ($\mathbf{\Sigma}^j$) to compute controlled variable K^j.
3. Initialize loop on segments ($i=0$, where i – current segment index, $i=0, 1, \ldots, N$).
4. Check the quality of the current segmentation E_{il}^j of the *i*-th segment, that is based upon charge density differences with neighboring segments. If $E_{il}^j > \varepsilon$, then go to step 5, else go to step 6.
5. Divide current segment *i* on *n* parts.
6. Go to next segment ($i=i+1$).
7. If there are no more segments go to step 8 else go to step 4.
8. For $j=j+1$ solve equation (1) and use its solution ($\mathbf{\Sigma}^j$) to find controlled variable K^j.
9. If $|K^j-K^{j-1}|/K^{j-1}>tol$, then go to step 3.

This algorithm is shown on Fig. 3 as a flowchart. To check the quality of the current segmentation we use the expression

$$E_{il}^j = m\mathbf{\Sigma}_{il}^j - \sum_{k\in I_l^j} \mathbf{\Sigma}_{kl}^j > \boldsymbol{\varepsilon} \tag{2}$$

where Σ_{il}^j – value of the charge density on segment *i* of conductor *l*, I_i^j – list of segments neighbouring with segment *i*, *m* – number of boundary segments. There are 2 types of checking the quality of the current segmentation (check 1 and check 2), qualities of these checking types are shown of Fig. 4. When doing segmentation of dielectric-dielectric boundaries we use needed elements of the first column of the **Σ** (Σ_{i1}^j) matrix.

The developed algorithm was implemented in TALGAT system [4] and tested on the example of stack of printed circuit board (PCB) shown in Fig. 5 [3]. Its geometric parameters are taken from the real PCB fragment: width of the central conductor w=890 µm, side conductors width is 5w (conductors are numbered from up to down and from left to right), gaps are s_1=500 µm, s_2=1890 µm, width of the central conductor and other continuous conductors is t=35 µm, prepreg width is h_1=h_3=144 µm, substrate thickness is h_2=220 µm. Solder mask thickness is h_M=30 µm. We use as a controlled variable (K) the entry of electrostatic inductance matrix (C, pF/m) that fits needed conductor. We consider several configurations of stacks (side conductors are grounded or aren't grounded) and computational variants (Table I).

TABLE I. STRUCTURE CONFIGURATION AND COMPUTATIONAL VARIANTS

Configuration	Side conductors (1, 3–7)	Controlled variable, pF/m
1	Grounded	C_2
2	Not grounded	C_2
3	Not grounded	C_6

III. NUMERICAL EXPERIMENTS

For a computational experiment we used a personal computer with the following parameters: platform – Intel(R) Core(TM) i7 CPU 970; processor frequency – 3.20 GHz; RAM size – 24 GBytes; number of cores – 6; virtual cores – 12; operating system – Windows 7x64; compiler – Microsoft Visual C++ 2013.

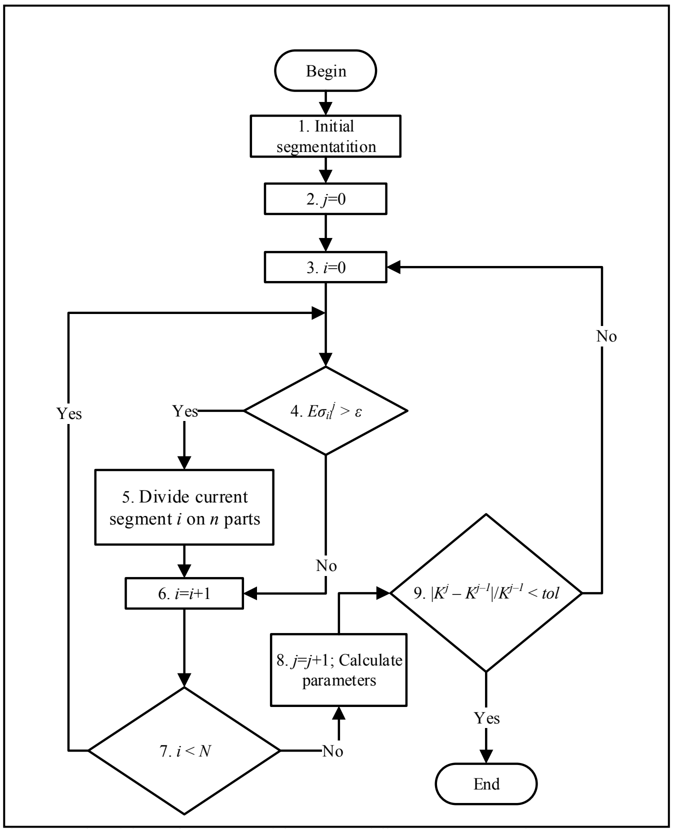

Fig. 3. Algorithm of AICOS with non-equidistant segmentation

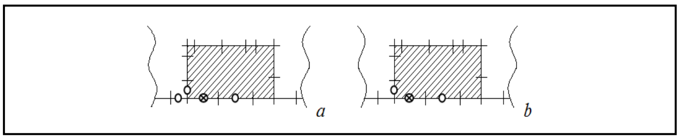

Fig. 4. Segments (○), neighbouring with segment (⊗): variants 1 (*a*) и 2 (*b*)

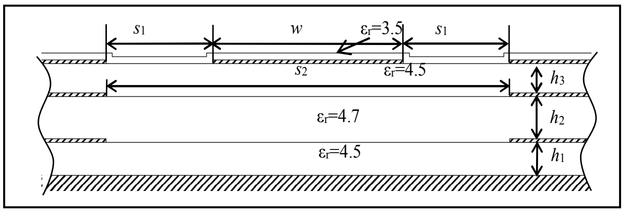

Fig. 5. Zoomed in fragment of the cross section of the PCB

A comparison of the developed algorithm of AICOS with non-equidistant segmentation and AICOS with equidistant segmentation [4] is performed. To solve equation (1) we used the LU-factorization and the subsequent solution with each right-hand side. The results are summarized in Tables II–VI where N is the number of segments at the j-th iteration, T is the iteration time. The following parameters of the algorithm of non-equidistant segmentation were used in the calculations: n = 2; ε = 0.01, 0.05, 0.1; *tol* = 0.001. For the algorithm that uses equidistant segmentation *tol* = 0.001. When calculating with the manually specified equidistant segmentation of 10 μm (we choose it supposing that we divide the boundaries of the conductors at the ends by at least 3 segments), the order of the linear systems matrix was N = 7135 for all configurations, with C_2=2315,63 and C_6=129,21 pF/m.

It can be seen from the data presented that both versions of the proposed algorithm are efficient and allow reducing the computational costs with acceptable accuracy. However, for different configurations, the different behavior has been obtained, so further research is required to identify their optimal parameters.

TABLE II. AICOS USING EQUIDISTANT SEGMENTATION [4], THE RESULTS OF THE PCB STACK ANALYSIS

j	**AICOS using equidistant segmentation**					
	Configuration 1 and 2			***Configuration 3***		
	N	***C*₂**	***T, s***	***N***	***C*₆**	***T, s***
0	105	120.45	0.01	105	2299.91	0.01
1	182	122.87	0.03	182	2305.27	0.03
2	340	123.92	0.12	340	2309.97	0.12
3	659	124.29	0.63	659	2312.37	0.63
4	1297	126.05	4.00	1297	2313.56	4.00
5	2568	127.07	28.78	–	–	–
6	5148	127.02	216.69	–	–	–

TABLE III. AICOS USING NON-EQUIDISTANT SEGMENTATION WITH CHECK TYPE 1, THE RESULTS OF THE CONFIGURATIONS 1 AND 2 ANALYSIS

	ε=0.01			**ε=0.05**			**ε=0.1**		
	N	***C*₂**	***T. s***	***N***	***C*₂**	***T. s***	***N***	***C*₂**	***T. s***
0	94	124.03	0.01	94	124.03	0.01	94	124.03	0.01
1	186	125.44	0.04	184	125.44	0.04	184	125.44	0.04
2	372	125.56	0.16	356	125.14	0.15	354	125.14	0.15
3	–	–	–	625	125.98	0.58	611	125.69	0.55
4	–	–	–	1047	126.68	2.22	999	126.47	2.04
5	–	–	–	1561	126.84	7.41	1437	126.91	6.31
6	–	–	–	2115	126.85	21.15	1855	126.85	15.3

TABLE IV. AICOS USING NON-EQUIDISTANT SEGMENTATION WITH CHECK TYPE 2, THE RESULTS OF THE CONFIGURATIONS 1 AND 2 ANALYSIS

j	**ε=0.01**			**ε=0.05**			**ε=0.1**		
	N	***C*₂**	***T. s***	***N***	***C*₂**	***T. s***	***N***	***C*₂**	***T. s***
0	94	124.03	0.01	94	124.03	0.01	94	124.03	0.01
1	186	125.44	0.04	184	125.44	0.04	184	125.44	0.04
2	370	125.56	0.15	350	125.34	0.14	348	125.36	0.14

TABLE V. AICOS USING NON-EQUIDISTANT SEGMENTATION WITH CHECK TYPE 1, THE RESULTS OF THE CONFIGURATION 3 ANALYSIS

j	**ε=0.01**			**ε=0.05**			**ε=0.1**		
	N	***C*₆**	***T. s***	***N***	***C*₆**	***T. s***	***N***	***C*₆**	***T. s***
0	94	2299.87	0.01	94	2299.87	0.01	94	2299.87	0.01
1	186	2292.61	0.03	184	2303.11	0.03	184	2303.11	0.03
2	372	2296.58	0.15	–	–	–	–	–	–
3	737	2305.02	0.79	–	–	–	–	–	–
4	1378	2311.93	4.51	–	–	–	–	–	–
5	2254	2314.89	20.10	–	–	–	–	–	–
6	3781	2316.02	90.92	–	–	–	–	–	–

TABLE VI. AICOS USING NON-EQUIDISTANT SEGMENTATION WITH CHECK TYPE 2, THE RESULTS OF THE CONFIGURATION 3 ANALYSIS

j	**ε=0.01**			**ε=0.05**			**ε=0.1**		
	N	***C_6***	***T. s***	***N***	***C_6***	***T. s***	***N***	***C_6***	***T. s***
0	94	2299.87	0.02	94	2299.87	0.01	94	2299.87	0.01
1	186	2292.61	0.04	184	2303.11	0.03	184	2303.11	0.03
2	370	2293.58	0.16	–	–	–	–	–	–
3	733	2305.02	0.79	–	–	–	–	–	–
4	1372	2311.93	4.36	–	–	–	–	–	–
5	2243	2314.86	20.21	–	–	–	–	–	–
6	3760	2316.09	89.02	–	–	–	–	–	–

IV. CONCLUSION

An AICOS algorithm with non-equidistant segmentation that is used in the analysis of strip structures is proposed. On the example of the analysis of a fragment of a PCB, a computational experiment showing the efficiency of the algorithm is performed.

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