Multiple Solution of Linear Algebraic Systems by Iterative Methods in the Analysis of Modal Filters

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***Abstract*— The paper investigates the dependence existing between the time of a multiple calculation of a capacitive matrix for new protective devices – modal filters, by the method of moments. The influence of order (direct or reverse) in which its geometric parameter is changed with frozen and seed preconditioners is considered. For computer-aided design of a modal filter we are considered the multiple solution of linear algebraic systems with dense matrix and several right-hand sides by BiCGStab, Block BiCGStab and Seed BiCGStab iterative methods. For calculations Matlab and TALGAT systems are used. The preferability of using the Block BiCGStab method and the seed preconditioner for analyses of modal filters are shown.**

Keywords*— *Modal filter, multiple solution, linear algebraic system, iterative method, capacitance matrix.

I. Introduction

To ensure electromagnetic compatibility (EMC) of radioelectronic equipment and to protect it from interference, various design solutions and devices are used. One such device is a modal filter (MF) based on the use of modal distortions in interconnects of multiconductor transmission lines. Taking this peculiarity into account in their design, it is preferable to use computer-aided design based on the application of a quasistatic approach and the method of moments. Indeed, in order to investigate similar structures, the numerical simulation is widely used. Simulation of electrical characteristics is often carried out using the electromagnetic analysis. However, for long 2D-structures a quasi-static approach is often relevant. This requires the solution of Poisson's equation. Particularly, it allows obtaining causal results taking into account frequency dependent losses in conductors and dielectrics. TALGAT software [1] is designed for computer simulation of a wide class of EMC problems by performing the following main functions: quasi-static analysis (calculation of matrices) of arbitrary 2D and 3D structures of conductors and dielectrics; electromagnetic analysis of arbitrary 3D wire structures; computation of time and frequency responses of multiconductor transmission lines; structural and parametric optimization. A coupled line is a basis for many structures, including the MFs. It is assumed in the analysis that a coupled line is uniform along its length with an arbitrary cross section. In the general case, the cross section with *N*cond signal conductors and a reference one is represented by the following *N*cond×*N*cond matrices of line per-unit-length parameters: inductance (**L**), coefficients of electrostatic induction (**C**), resistance (**R**), conductance (**G**). In paper [2] an approach based on a modified nodal admittance matrix has been presented for the formulation of network equations including the coupled transmission line, terminal, and interconnecting networks. Voltages in the time domain are obtained by applying the inverse fast Fourier transform. Matrices **L**, **C** and **G** are calculated by a method of moments.

Investigation was carried out at the expense of the Ministry of Education and Science of Russian Federation project No. 8.9562.2017/8.9.

In order to find the parameters of interest when modal filters analyzing, it is required to solve subsequently the linear systems of the form

$$\mathbf{A}_k\mathbf{X}_k=\mathbf{B} \qquad (1)$$

where **A** – dense, square and nonsymmetric matrix of order *N*, **B** – $N\times N_{\mathrm{COND}}$ matrix of different right-hand sides, N_{COND} – number of conductors in structure, k=1, 2, …, m, m – number of linear systems. In this case, systems (1) can be solved by LU-decomposition and iterative methods with all righ-hand sides separately or block versions of Krylov type iterative methods.

In paper [3] the dependence existing between the time of a multiple solution of a capacitive matrix of the strip structures and the order (direct or reverse) in which its geometric parameter is changed was investigated. To accelerate solution of sequence of the (1) with one right-hand side, iterative method was used during this computation. Acceleration has been obtained due to selection of the reverse order.

The purpose of this paper is to highlight new results of applying iterative methods for solving the systems (1) in the analysis of MF by quasistatic approach and the method of moments.

II. Selection of Iterative Methods

Traditionally, linear systems with a dense matrix are solved by means of Gaussian elimination (which has been known for more than 2000 years) [4] or its compact model, based on LU-decomposition. For example, to calculate the capacitance matrix derived from the same structure when changing the value of the permittivity of dielectrics the block

LU decomposition is used [5, 6]. This algorithm is based on the fact that the diagonal entries in the right lower corner of the matrix vary only. However, in the general case, changes in the sizes of a structure cause variations in irregularly located matrix entries [7], so it is preferable to use iterative methods to accelerate solutions. For example, the iterative method (BiCGStab), used to accelerate the solution with one right-hand side, showed a significant speed up with respect to Gaussian elimination [8]. To accelerate the iterative process the two methods were used: the initial guess of the current linear algebraic system is the computed solution of the previous one; use for current system solution the implicit preconditioning matrix **M**, computed from the first coefficient matrix. However, the effectiveness of preconditioning decreases with increasing difference between the first and a current matrix. To solve this problem it is suggested to recompute the matrix **M** when the rate of convergence in solving the current linear system is too slow [9]. For this solution the existence of the optimal threshold value (wherein the time of linear system solution is minimal) is shown. However, it is not possible to determine a priori when to recompute the matrix **M**. Thus, a search a priori condition of the recomputation is relevant. Other conditions have been investigated in paper [10], so they are omitted in this paper.

When the block versions of Krylov type iterative methods are used then the subspace is $K_m(\mathbf{A}, \mathbf{R}) = span\{\mathbf{R}, \mathbf{AR}, \mathbf{A}^2\mathbf{R}, \ldots, \mathbf{A}^{m-1}\mathbf{R}\}$, where $\mathbf{R}$ – initial residual matrix of $N\times m$ size. Matrix bases of subspaces K and L are: $\mathbf{V} = [\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_m]$ and $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_m]$, where $\mathbf{V}$, $\mathbf{W}$ – matrices of $N\times m$ size. Thus, almost any projection iterative method can be adapted to solve such tasks. For solution of sparse matrices, there are the following methods: Block BiCGStab [11], Block GMRES [12] etc. [13].Typically, if an effective preconditioner is used, block methods are preferred rather than sequential solution of linear systems with different right-hand sides. However, if all right-hand sides are unavailable at the same time, these methods are not applicable. Anyway, one can use the method proposed in [14]. It looks for a starting vector in the space spanned by the previous solution vectors in the sequence, which is helpful if the solution vectors are correlated.

In this paper, as the iterative methods we used BiCGStab methods, because they showed a good performance for dense linear system with one right-hand side solution [15].

III. Selection of Preconditioner

Preconditioning is necessary to ensure fast convergence of the iterative method. There are several approaches to solving of the sequence (1) by an iterative method, when the system matrix is sparse.

The first approach is based on recomputation of a preconditioner from scratch for each matrix of sequence. It is obvious that such an approach has the highest computational costs. The second approach is based on the computation of the preconditioner from the first matrix of sequence and its using for the solution of other systems (frozen preconditioner) [16]. The third approach lies in update of a preconditioner obtained from the matrix of one of the systems (seed preconditioner), and in the repeating the update when necessary [17]. The fourth approach is based on periodic recomputation of a preconditioner before the solution of each p-th system and use of the preconditioner as a frozen preconditioner during the period. The fifth approach is equivalent to the previous one, however, if during the period the number of iterations required for solutions of the current system ($iter_j$) is bigger than the sum of number of iterations required for the solutions of the first system in period ($iter_0$) and a predetermined margin (s), i.e. $iter_j > iter_0 + s$, $j=1, \ldots, p-1$, then the preconditioner is updated [18]. The last method is based on adaptive use of information about the Krylov subspaces obtained on the previous steps, used to update the preconditioner and the iterative method (recycling of Krylov subspaces) [19].

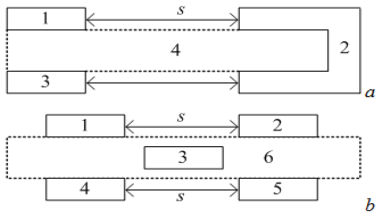

Fig. 1. Cross section of the examined structures: 1 – symmetrical MF with broad-side coupling (1–3 – conductors, 4 – dielectric) (*a*); 2 – reverse symmetrical MF (1–5 – conductor, 6 – dielectric) (*b*)

For the first study in this paper we used frozen preconditioner with different order of solution (direct and reverse), seed preconditioner and full LU-decomposition for its forming. Using of full LU-decomposition is a best case of the sequence (1) with one right-hand side solution [8, 9].

IV. Numerical Experiments

For numerical experiments we used a personal computer with the following parameters: platform – Intel(R) Core (TM) i7 CPU 970; processor frequency – 3.20 GHz, memory – 24 Gb; number of cores – 6; operating system – Windows 7x64; Matlab 2013b.

For two structures (Fig. 1) 100 matrices were formed in TALGAT software [**Error! Bookmark not defined.**]. For the structure 1 (Fig. 1*a*), which is a modal filter with broad-side coupling [20], the matrices of order N=3001 were obtained by varying the gap (s) in the range of 100, 101 ... 200 μm. Number of right-hand sides was 3 and number of conductor segments was 2550. For structure 2 (Fig. 1*b*), being a mirror MF [21], matrixes with N=3109 were obtained by means of change of spaces (s) in the range of 16.9, 16.8 … 7.1 μm. Number of right-hand sides was 5 and number of conductor segments was 2220. We compared the performance of the Block BiCGStab method [11], the Seed BiCGStab [22] method and the BiCGStab method applied to each single right-hand side (separate solution). We also used LU decomposition as a frozen preconditioner (direct and reverse order) and a seed preconditioner (formed from $\mathbf{A}_{50}$). Iterations were continued until the relative residual norm became smaller than 10^{-6}. Maximum number of iterations for all methods was 200.

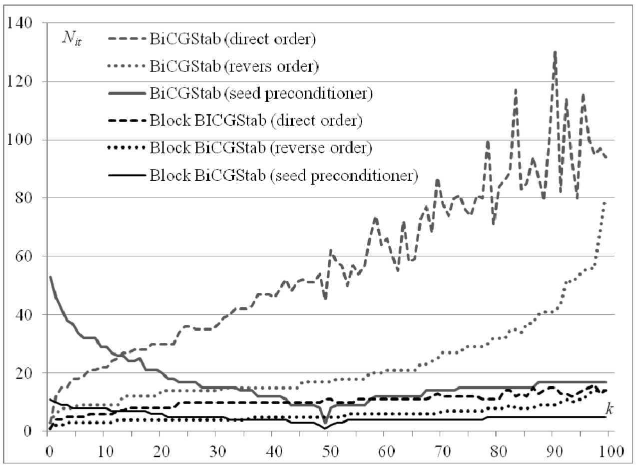

Fig. 2. Number of iterations for the BiCGStab and the Block BiCGStab methods with zero initial guess for the structure 1

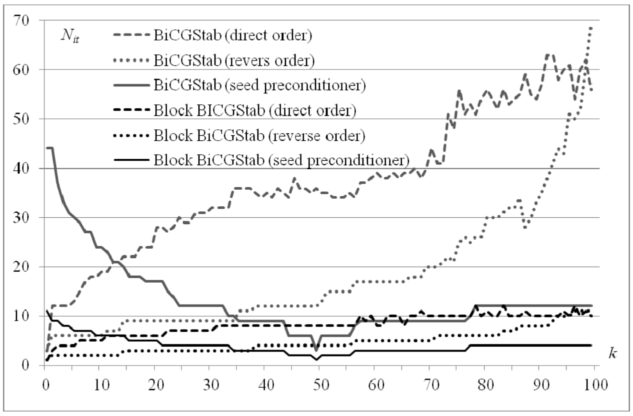

Fig. 3. Number of iterations for the BiCGStab and the Block BiCGStab methods with previous solution as an initial guess for structure 1

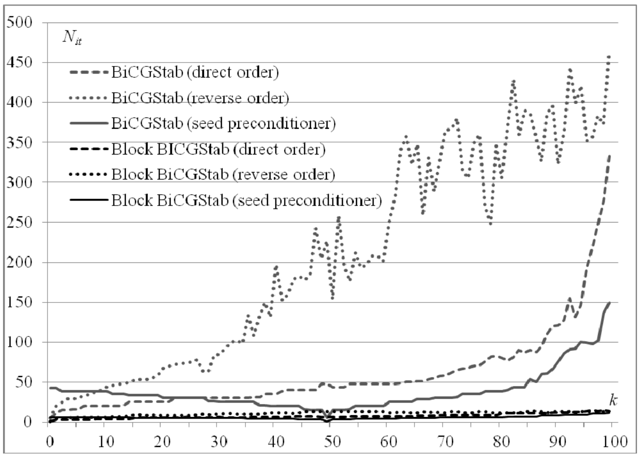

Fig. 4. Number for iterations by the BiCGStab and the Block BiCGStab methods with zero initial guess for the structure 2

Obtained number of iterations by the BiCGStab and the Block BiCGStab methods with zero initial guess ($\mathbf{X}_0$=0) for the structure 1 are shown in Fig. 2. Results for previous solution used as an initial guess ($\mathbf{X}_0$=$\mathbf{X}_{k-1}$) are shown in Fig. 3. Similar results for the structure 2 are shown in Fig. 4 and Fig. 5. In Table I we listed the solution time ratios with respect to the solution with the BiCGStab method and zero initial guess for the structure 1. It can be seen that the use of the seed preconditioner is preferable. The use of the reverse order of the solution with respect to the direct also allows increasing the productivity. Also for all the cases considered, the use of the previous solution as the initial guess of the next system is preferable to the zero initial guess.

TABLE I. SPEED UP OF 100 LINEAR SYSTEMS SOLUTION BY THE BICGSTAB AND THE BLOCK BICGSTAB METHODS WITH DIFFERENT PRECONDITIONERS FOR THE STRUCTURE 1

Preconditioner		BiCGStab		Block BiCGStab	
		$\mathbf{X}_0$=0	$\mathbf{X}_0$=$\mathbf{X}_{k-1}$	$\mathbf{X}_0$=0	$\mathbf{X}_0$=$\mathbf{X}_{k-1}$
Frozen	Direct order	1.00	1.55	3.47	4.34
	Reverse order	2.61	3.71	6.36	8.06
Seed		3.39	4.77	7.22	9.01

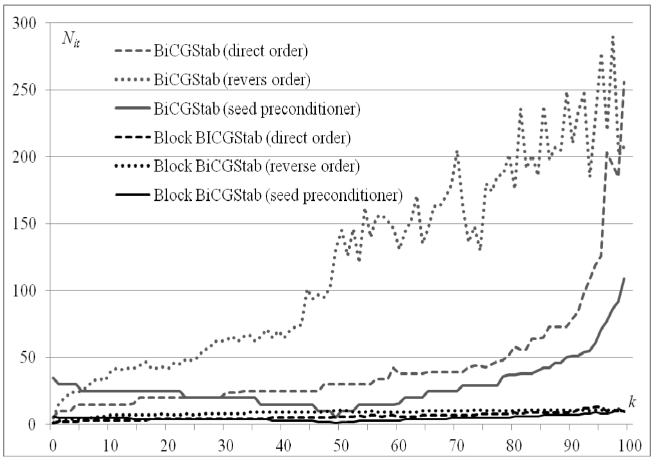

Fig. 5. Number of iterations for the BiCGStab and the Block BiCGStab methods with previous solution as an initial guess for the structure 2

In Table 2 we listed similar ratios for the structure 2. As for structure 1, it is preferable to use the seed preconditioner, and to use previous solution as the initial guess of the next system. At the same time the use of the reverse order with respect to the direct one, on the contrary, allows to obtain lower productivity.

TABLE II. SPEED UP OF 100 LINEAR SYSTEMS SOLUTION BY THE BICGSTAB AND THE BLOCK BICGSTAB METHODS WITH DIFFERENT PRECONDITIONERS FOR THE STRUCTURE 2

Preconditioner		BiCGStab		Block BiCGStab	
		$\mathbf{X}_0$=0	$\mathbf{X}_0$=$\mathbf{X}_{k-1}$	$\mathbf{X}_0$=0	$\mathbf{X}_0$=$\mathbf{X}_{k-1}$
Frozen	Direct order	1.00	1.40	3.88	5.28
	Reverse order	0.26	0.53	2.86	3.21
Seed		1.57	2.05	5.44	5.88

Next, we evaluated the use of the Seed BiCGStab method. The method was not efficient for the task, because, regardless of the approach used, the maximum number of iterations was required for solving some systems. For example, the number of iterations required for the analysis of the structures under consideration (best and worst cases) is shown in Fig. 6. It can be seen that the obtained dependences are distinguished by a large oscillation.

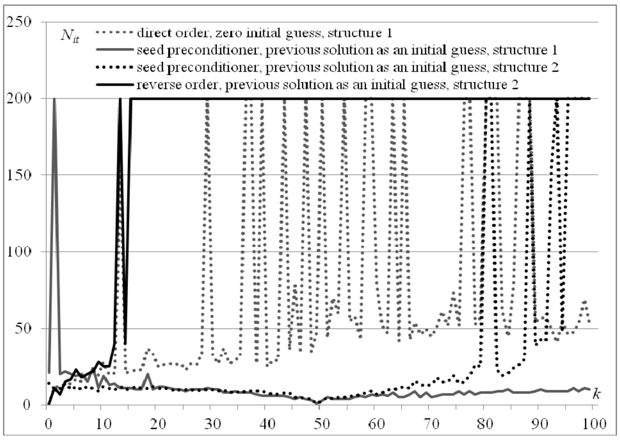

Fig. 6. Number of iterations for the seed BiCGStab method

V. Conclusion

The process of multiple solution of linear systems with different right-hand sides by iterative methods arising from the analysis of protecting devices by method of moments is shown. The performance of the Block BiCGStab, Seed BiCGStab method and BiCGStab methods with a frozen preconditioner and a seed preconditioner and two types of initial guess were compared. The preferability of using the Block BiCGStab method, the seed preconditioner and previous solution as an initial guess for next system is shown. Acceleration up to 9 times has been obtained due to selection of the Block BiCGStab method and seed preconditioner with respect to the BiCGStab method and frozen preconditioner. Also it is shown that the use of Seed BiCGStab method for analysis of modal filters is not efficient.

References

[1] S.P. Kuksenko, T.R. Gazizov, A.M. Zabolotsky, R.R. Ahunov , R.S. Surovtsev, V.K. Salov, Eg.V. Lezhnin, “New developments for improved simulation of interconnects based on method of moments,” Int. conf. on modelling, simulation and applied mathematics, 2015, pp. 1–8.

[2] J.R. Griffith, M.S. Nakhla, “Time-domain analysis of lossy coupled transmission lines,” IEEE Trans. on Microwave Theory and Techniques, 1990; vol. 38, is. 10, pp. 1480–1487.

[3] R.R. Akhunov, S.P. Kuksenko, T.R. Gazizov, “A simple method to speed up the computation of capacitive matrixes of strip structure when changing its geometric parameter,” Dokladi TUSUR, 2015, vol. 4, is. 38, pp. 41–44. (in Russian)

[4] J.F. Grcar, “Mathematicians of Gaussian elimination,” Notices of the AMS, 2011, vol. 58, is. 6, pp. 782–792.

[5] R.S. Surovtsev, V.K. Salov, “Investigation of acceleration of multiple solution of linear algebraic systems with a partially changing matrix by a block method,” Electromagnetic waves and electronic systems, 2012, vol 17, is. 10, pp. 22–24. (in Russian).

[6] R.S. Surovtsev, S.P. Kuksenko, “Computation of the capacitance matrix of an arbitrary system of conductors and dielectrics by the method of moments: The dependence of the speed up owing to using block LU factorization on the order of the coefficient matrix,” Izvestiya Vuzov Fizika, 2012, vol. 55, is. 9/3, pp. 126–130. (in Russian)

[7] T.R. Gazizov, “Analytic expressions for Mom calculation of capacitance matrix of two dimensional system of conductors and dielectrics having arbitrary oriented boundaries,” IEEE EMC Symposium, 2001, pp. 151–155.

[8] R.R. Akhunov, S.P. Kuksenko, T.R. Gazizov, “Acceleration of multiple iterative solution of linear algebraic systems in computing the capacitance of a microstrip line in wide ranges of its sizes,” Journal of mathematical sciences, 2015, vol. 207, is. 5, pp. 686–692.

[9] R.R. Akhunov, S.P. Kuksenko, T.R. Gazizov, “Multiple solution of systems of linear algebraic equations by an iterative method with recomputed preconditioners,” Journal of mathematical sciences, 2015, vol. 207, is. 5, pp. 693–697.

[10] T.R. Gazizov, S.P. Kuksenko , R.R. Akhunov, ”Acceleration of multiple solution of linear systems for analyses of microstrip structures,” International journal of mathematical models and methods in applied sciences, 2015, vol. 9, pp. 721–726.

[11] A. El Guennouni, K. Jbilou , H. Sadok, “Block version of BiCGSTAB for linear systems with multiple right-hand sides,” Electronic Transactions on Numerical Analysis, 2003, vol. 16, pp. 129–142.

[12] B. Vital, Etude de quelques methodes de resolution de problemes lineaires de grande taille sur multiprocesseur, Ph.D. thesis, Universite de Rennes, Rennes, France, 1990.

[13] V. Puzyrev , J.M. Cela, “A review of block Krylov subspace methods for multisource electromagnetic modeling,” Geophysical Journal International, 2015, vol. 202, pp. 1241–1252.

[14] P.F. Fischer, “Projection techniques for iterative solution of Ax = b with successive right-hand sides,” Computer Methods in Applied Mechanics and Engineering, 1998, vol. 163, pp. 193–204.

[15] S.P. Kuksenko, T.R. Gazizov, “Dense linear system solution by preconditioned iterative methods in computational electromagnetics,” 19th Int. Zurich symp. electromagnetic compatibility, 2008, pp. 918–921.

[16] A. Meister , C. Vömel, “Efficient preconditioning of linear systems arising from the discretization of hyperbolic conservation laws,“ Advances in Computational Mathematics, 2001, vol. 14, is. 1, pp. 49–73.

[17] M. Benzi , D. Bertaccini, “Approximate inverse preconditioning for shifted linear systems,” BIT Numerical Mathematics, 2003, vol. 43, is. 2, pp. 231–244.

[18] P. Birken , J.D. Tebbens, A. Meister, M. Tuma, “Preconditioner updates applied to CFD model problems,” Applied Numerical Mathematics, 2008, vol. 58, is. 11, pp. 1628–1641.

[19] J. Baglama , D. Calvetti , G.H. Golub , L. Reichel, “Adaptively preconditioned GMRES algorithms,” SIAM Journal on Scientific Computing, 1998, vol. 20, is. 1, pp. 243–269.

[20] A.M. Zabolotsky, T.R. Gazizov, A.O. Melkozerov, P.E. Orlov, E.S. Dolganov, “Improved design of modal filter for electronics protection,” IEEE International Conference on Lightning Protection, 2012, pp. 1–4.

[21] A.M. Zabolotsky, “Application of reflective symmetry for modal filtration improvement,” Dokladi TUSUR, 2015, vol. 2, is. 36, pp. 41–44. (in Russian)

[22] K. Jbilou,” A Note on the Block and Seed BiCGSTAB Algorithms for Nonsymmetric Multiple Linear Systems,” American Journal of Algorithms and Computing, 2016, vol. 3, is. 1, pp. 1–13.