A Simplified Algorithm for Calculating the Capacitance of a Microstrip Line Segment Considering End Effects

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***Abstract*— This paper presents an algorithm for the simplified calculation of the capacitance of a microstrip line segment considering end effects, based on two-dimensional charge distribution modeling instead of three-dimensional modeling. The algorithm is verified for a wide range of geometric parameters: conductor width from 1 um to 10 mm, thickness from 10 to 100 um, substrate height from 1 mm to 10 mm, and line length from 10 um to 10 cm. It is established that the accuracy of the calculation results is determined by the ratio of the line length to its width, *l*/*w*. For *l*/*w*>100, the relative error does not exceed 1.5%; for 10<*l*/*w*≤ 100, it ranges from 1.5–6%; and for *l*/*w*≤10, it increases to 45%, which defines the applicability limits of the method. It is shown that accounting for the end faces of the transmission line conductors using the proposed algorithm reduces the relative difference by 2–4 times compared to calculations without considering the ends. The obtained results can be used in the design of high-frequency electronic equipment units for quick estimation of conductor capacitance, particularly during optimization.**

Keywords— microstrip line capacitance, 2D modeling, 3D modeling, end effects, method of moments.

I. Introduction

Transmission lines are among the most common structures used in modern electronic equipment, employed in integrated circuits, printed circuit boards, and various high-frequency modules [1], [2]. Accurate determination of the capacitance of transmission line conductors is an important task in electronic equipment design, as it affects the characteristic impedance, signal propagation delay, losses, and the overall electromagnetic compatibility (EMC) of the equipment [3], [4].

Modern software tools based on the method of moments (MoM) allow for highly accurate calculation of three-dimensional electromagnetic structures [5]. However, 3D-modeling is characterized by significant computational costs, both in terms of time and required personal computer resources. This becomes especially critical during parametric optimization, where it is necessary to repeatedly calculate the characteristics of a structure while varying various geometric parameters. The execution time for a single calculation can reach tens of seconds or even minutes, making the optimization of printed circuit boards with numerous microstrip lines very lengthy [6], [7].

An alternative to 3D-modeling is the use of two-dimensional models, which allow for rapid calculation of the per-unit-length parameters of an infinitely long line [8]. However, for a microstrip line of short length, this approach has a significant drawback: it does not account for end effects. For long lines, the contribution of end effects to the total capacitance is negligible, but for miniature lines where the line length becomes comparable to its transverse dimension, these effects can introduce significant error [9], [10]. Consequently, there is a need for sufficiently accurate accounting of end effects.

Thus, there is a need to account for end effects with high computational efficiency.

Therefore, the development of a hybrid approach that combines the high speed of 2D-modeling with the ability to account for end effects due to the finite conductor length is relevant.

The aim of this work is to develop an algorithm for calculating the capacitance of a microstrip transmission line, represented by its cross-section, while taking into account end effects.

II. Capacitance Calculation Algorithm

A. Basic Calculation Relations

The main idea of the proposed algorithm is to represent the capacitance of a finite-length transmission line as the sum of two components: the per-unit-length capacitance and the capacitances of the two end faces. This representation is physically justified because for a sufficiently long line, the electric field distribution does not change along the line's length, and the contribution to the capacitance of each end face is localized near its respective end.

Then, the total capacitance of a line segment of length l can be calculated as:

$$C_{\text{3D2D}} = C_{\text{2D}}\, l + 2C_{\text{end}}, \tag{1}$$

where C_{3D2D} is the capacitance calculated by the proposed algorithm; C_{2D} is the per-unit-length capacitance of the line, calculated based on the geometric parameters of the cross-section, for example, using the MOM2D module of the

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TUSUR.EMC software [11]; l is the line length; C_{end} is the capacitance of one of the end faces. In (1), C_{end} is doubled because the three-dimensional model of the line has two end faces of width w and height t (Fig. 1).

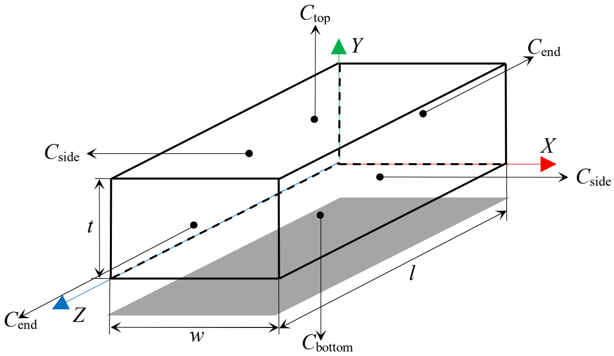

Fig. 1. Three-dimensional model of the transmission line.

The accuracy of the algorithm is determined by the method for calculating the end face capacitance C_{end}, which can be computed based on the charge distribution in the line's cross-section obtained from 2D-modeling.

B. Calculation of End Face Capacitance via Charge Distribution

For the cross-section of a microstrip line with conductor width w and thickness t, the MOM2D module of the TUSUR.EMC software provides a function for calculating the charge distribution in the segments of the structure's cross-section (Fig. 2). This function solves the electrostatic problem for the cross-section of an infinitely long line, allowing the determination of the surface charge distribution on the conductor. As a result, charges per unit line length are obtained for each segment.

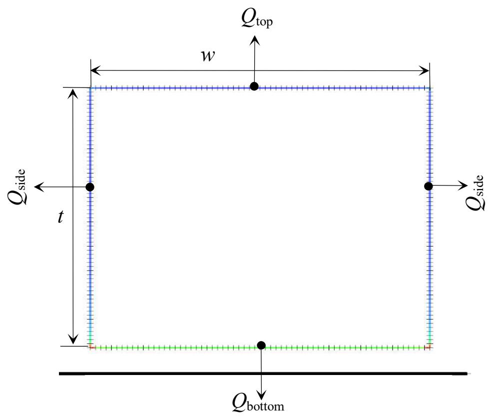

Fig. 2. Model of the line cross-section with charge density visualization.

The algorithm for calculating C_{end} includes the following steps:

1. Solving the electrostatic problem for the cross-section of the microstrip line, resulting in the determination of the charge per unit line length for each segment on the conductor surface

2. Identifying the segments belonging to the side face of the conductor, i.e., the face with thickness t (Fig. 1).

3. Summing the charges of the identified segments to obtain the total charge Q_{side} per unit line length.

4. The per-unit-length capacitance of the side face is calculated as:

$$C_{\text{side}} = \frac{Q_{\text{side}}}{U}. \qquad (2)$$

where U is the potential difference between the conductor and the ground plane.

5. Since the length of the end face is the conductor width w (Fig. 2), when transitioning to the three-dimensional structure, C_{end} is obtained by multiplying C_{side} by the conductor width:

$$C_{\text{end}} = C_{\text{side}}\, w. \qquad (3)$$

III. Algorithm Verification

A. Verification Methodology

To verify the adequacy of the proposed algorithm, a series of computational experiments was conducted. The results calculated using the algorithm C_{3D2D} were compared with the results of 3D-modeling C_{3D} performed in the MOM3D module of the TUSUR.EMC software. The accuracy criterion was the relative error δ, defined as:

$$\delta = \frac{|C_{3D} - C_{3D2D}|}{C_{3D}} \cdot 100\%. \qquad (4)$$

where C_{3D} is the capacitance obtained from 3D-modeling.

Verification was carried out over a wide range of geometric parameters, allowing patterns in the influence of these parameters on the algorithm's accuracy to be identified. The studied parameters were varied: conductor width w from 1 um to 10 mm; conductor thickness t from 10 to 100 um; distance to ground plane h from 1 um to 10 mm; conductor length l from 10 um to 10 cm. A total of over 500 different parameter combinations were considered.

B. Convergence of Results

Before the calculations, the convergence of the results with increasing segmentation density was assessed. Fig. 3 shows the dependence of the per-unit-length capacitance on the segment length into which the cross-section is divided. It can be seen that as the segment length decreases, the capacitance approaches a limiting value. Starting from $w/32$, the change in capacitance with further segment size reduction is less than 1%. To ensure high accuracy, a discretization of $w/64$ was used in this study.

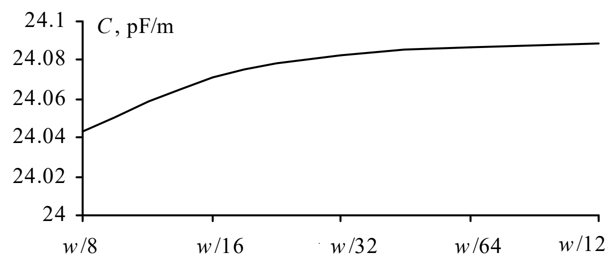

Fig. 3. Change in per-unit-length capacitance with increasing segmentation density.

C. Verification Results

Analysis of the obtained results revealed several key patterns. The most significant influence on the algorithm's accuracy is exerted by the ratio of the line length to its width (l/w). Table 1 presents calculation results for fixed parameters $w=t=h=10$ um with the line length varying from 10 um to 10 cm.

TABLE I. CAPACITANCE AND RELATIV ERRORS FOR DIFFERENT LINE LENGTHS

l, m	l/w	C_{3D}, fF	C_{3D2D}, fF	δ, %
10^{-5}	10^{0}	0.94	0.52	44.7
10^{-4}	10^{1}	4.19	3.67	12.4
10^{-3}	10^{2}	35.73	35.23	1.4
10^{-2}	10^{3}	350.07	350.81	0.2
10^{-1}	10^{4}	3505.55	3506.60	0.03

From Table 1, it is evident that at $l/w=100$, the relative error is 1.4%, and with increasing length, it decreases to 0.03%. At $l/w=10$, the error increases to 12.4%, and at $l/w\leq1$, it reaches 45%. Consequently, the proposed algorithm provides high accuracy (up to 1.4%) for $l/w>100$ and acceptable accuracy (up to 12.4%) for $10<l/w\leq100$.

Fig. 4 shows the dependence of δ on h for $w=t=l=10$, 100 um with and without accounting for end effects. It can be seen (Fig. 4) that the proposed algorithm reduces the error by a factor of 2 compared to the simplified approach.

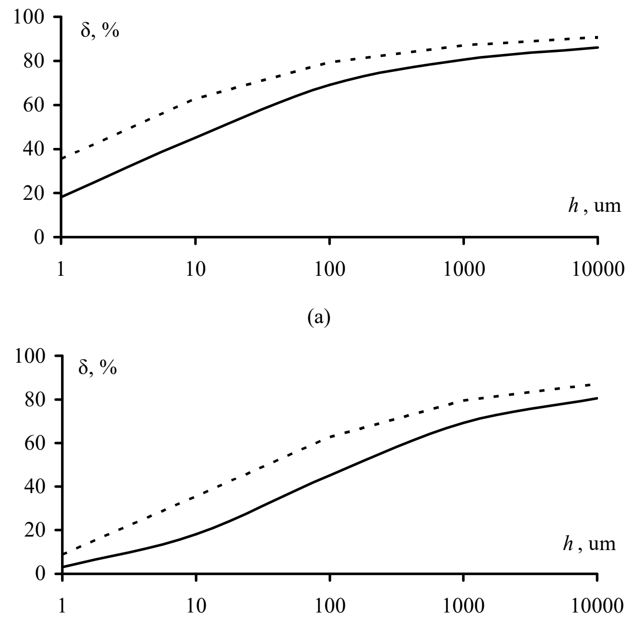

Fig. 4. Dependence of δ on h for $w=t=l=10$ um (a) and $w=t=l=100$ um (b) with (—) and without (- -) accounting for end effects.

IV. DISCUSSION OF RESULTS

A. Physical Reasons for Observed Patterns

The obtained results have a clear physical interpretation. For a long line ($l/w\gg1$), the electric field distribution in the central part of the line does not change along its length. In this approximation, the per-unit-length capacitance C_{2D} multiplied by the line length almost entirely determines the total capacitance of the transmission line, and the contribution of the end faces is negligible. Consequently, even a small error in determining C_{end} does not significantly affect the final result.

As the line length decreases, the situation changes. The contribution of the end faces increases because it is independent of length, while the contribution of the main part is proportional to l. When $l/w<10$, the contribution of the end faces becomes comparable to the contribution of $C_{2D}l$, leading to an increase in error when they are approximated.

B. Recommendations for Practical Application of the Algorithm

Based on the analysis performed, the following recommendations for the application of the proposed algorithm are formulated.

For transmission lines with $l/w>100$, δ<1.5%, which allows the algorithm to be used for solving most engineering tasks in the design of microwave devices.

For transmission lines with $10<l/w\leq100$, the algorithm can be used for estimation and parametric optimization. The expected δ will range from 1.5 to 6%, which is acceptable in most cases. However, for higher accuracy, calculations should be performed in the MOM3D module.

For transmission lines with $1<l/w\leq10$, the application of the algorithm is possible only for preliminary estimates, since δ=20-40%. In this range, it is recommended to use the algorithm's results as an initial approximation for subsequent optimization in the MOM3D module.

The influence of h on the accuracy of the results can also be explained by the fact that at small h, the conductor is located close to the ground plane, and the field is concentrated in the narrow gap between them. For $h/t>100$, the calculation accuracy decreases, especially for transmission lines with $l/w<100$. In such cases, it is recommended to reduce the acceptable l/w values by 2–3 times compared to those stated above.

V. CONCLUSION

This paper presents an algorithm for calculating the capacitance of a microstrip line accounting for end effects, according to which the total line capacitance is calculated as the sum of the product of the per-unit-length capacitance and the line length, plus twice the end face capacitance. Here, the end face capacitance is determined through the charge on the side face of the conductor in the 2D module, which allows the influence of geometric parameters on the charge distribution to be considered.

It is established that the accuracy of the algorithm depends most significantly on l/w: for $l/w>100$, δ<1.5%, indicating high accuracy of the algorithm for relatively long lines; for $10<l/w\leq100$, δ=1.5-6%, which is acceptable for most engineering tasks; for $l/w\leq10$, δ=20-45%, which limits the application of the algorithm.

It is shown that the proposed algorithm reduces the error by several times compared to the simplified approach that does not account for end effects.

The obtained results allow rapid capacitance estimation in the early stages of PCB design, especially during parametric sweeps where 3D simulation is too slow. Future work will focus on extending the proposed approach to coupled microstrip lines and more complex multilayer structures.

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